Fourier Representations of Switching Functions for Circuit Design

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TICSP Tampere International Center for Signal Processing

Outline

Motivations - Why compact representations?

Spectral Methods for Logic Design

Fourier transforms on groups

Complexity of Fourier representations

Switching Theory and Digital Signal Processing

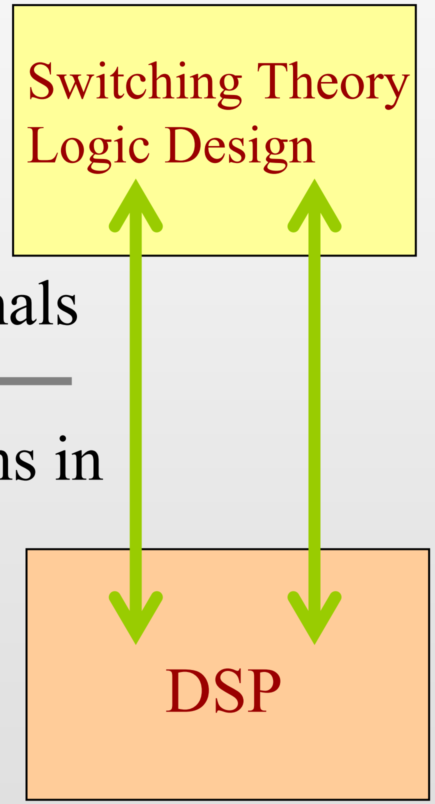
Switching theory mathematic foundations for Logic design

Transmission

Storage

Processing

of information encoded in digital (binary) signals



Methods in signal processing to solve problems in

Design

Optimization

Verification and testing

of switching circuits and systems

Goal of the Paper

Applications of group-theoretic methods in DSP to

Derivation of compact representations for switching functions

Logic circuit design from spectral representations

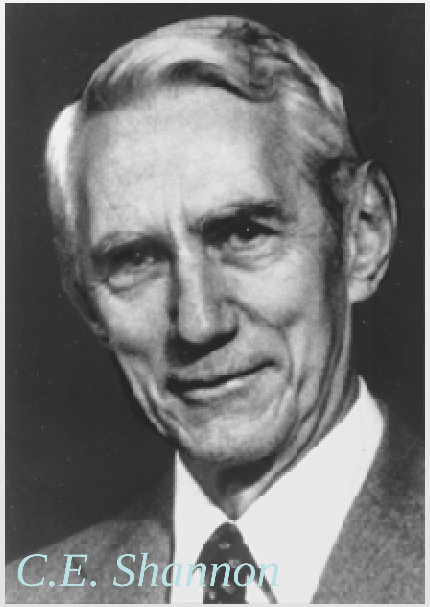
Fourier series expression with varyed domain groups

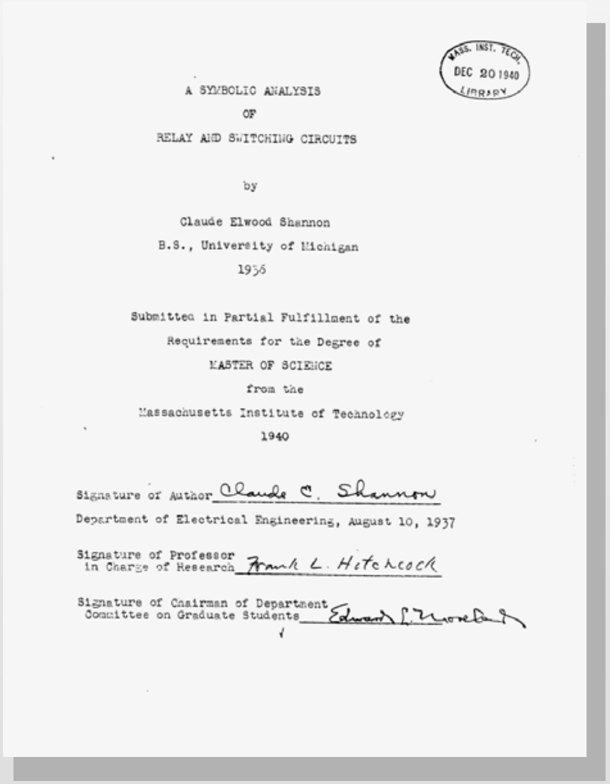
Estimation of complexity

Design of logic circuits with regular structure

Transmission of Information

Discrete Signals and Digital Systems





A SYMBOLIC ANALYSIS
OF
RELAY AND SWITCHING CIRCUITS

by

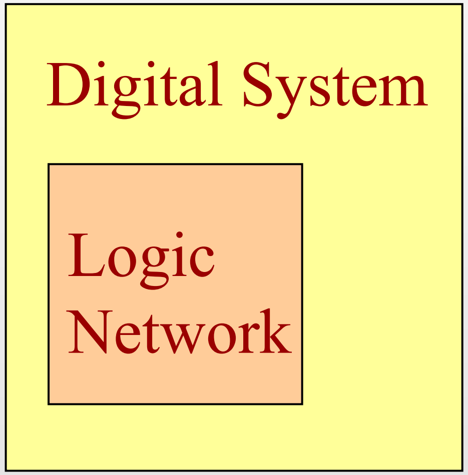
Claude Elwood Shannon
B.S., University of Michigan
1936

Submitted in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
from the
Massachusetts Institute of Technology
1940

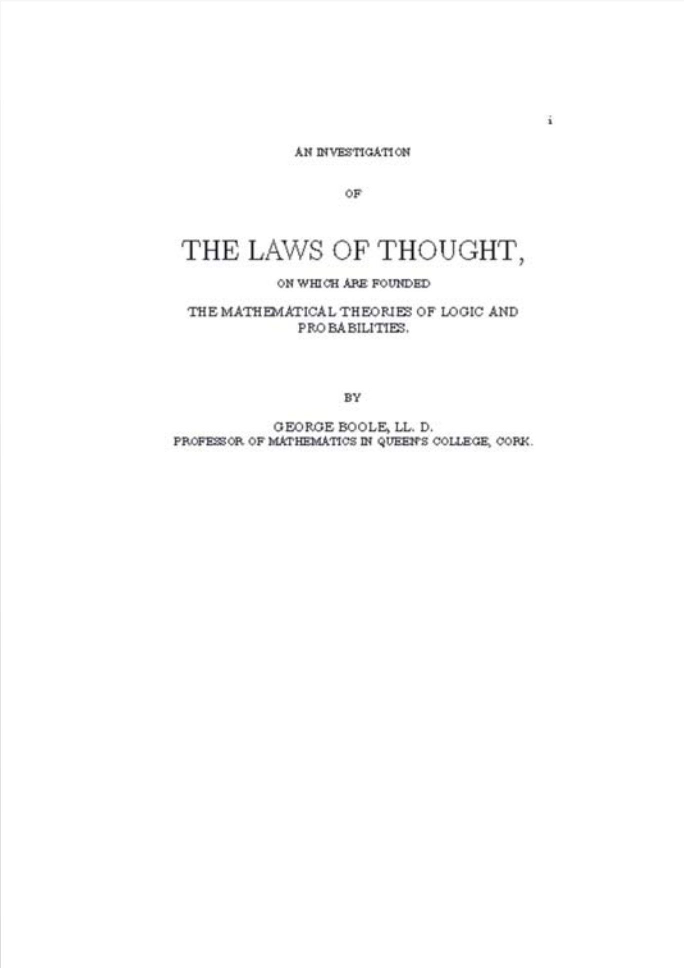
Signature of Author Claude E. Shannon
Department of Electrical Engineering, August 10, 1937

Signature of Professor
in Charge of Research Frank L. Hitchcock

Signature of Chairman of Department
Committee on Graduate Students



Boolean Algebra



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AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

BY

GEORGE BOOLE, LL. D.
PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

Mathematical Analysis of Logic

1847

1854

Design of digital systems from skills and art to science and engineering

Why Compact Representations?

System-on-Chip
Network-on-Chip

Requirements in practice

Design objective

Use fewer chips
Do more on a chip

Eliminating redundant gates

Fries up the chip area
Reduces power dissipation
Simplifies testing, etc.

Spectral Representations

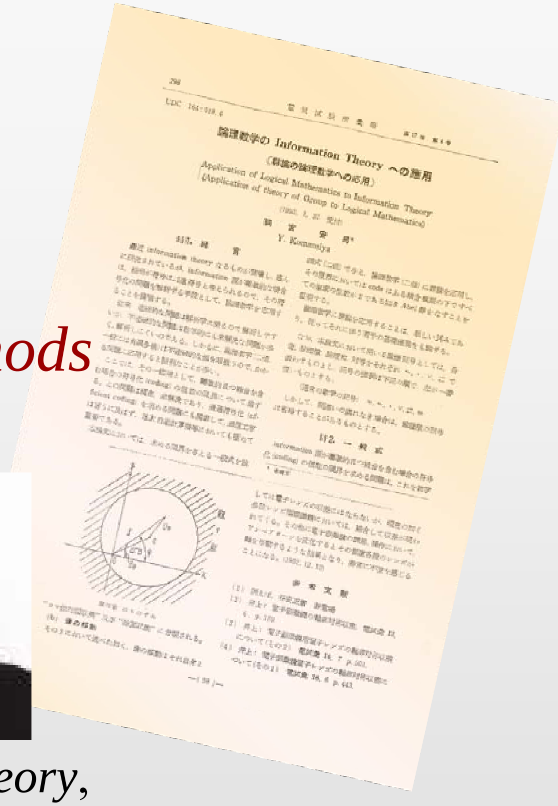
Compact encoding of information

Natural phenomena modelled by spectral methods

Implications

Many, for instance

Different algebraic structures



論理数学の Information Theory への應用

Application of Logical Mathematics to Information Theory

(Application of Theory of Group to Logical Mathematics)

Y. Komamiya

Group theory
Spectral techniques

Komamiya, Y., *Information Theory*,
Application of Logical Mathematics to Information Theory,
Application of Theory of Group to Logical Mathematics, 1953.

Hurst, S.L., *Logical Processing of Digital Signals*,
Crane Russak and Edward Arnold, London and Basel, 1978.

Aiken and his Comments

SYNTHESIS OF ELECTRONIC COMPUTING AND CONTROL CIRCUITS

BY

THE STAFF OF THE COMPUTATION LABORATORY

CAMBRIDGE, MASSACHUSETTS
HARVARD UNIVERSITY PRESS
1951

STAFF OF THE COMPUTATION LABORATORY

Howard H. Aiken

PREFACE

During the spring term of the academic year 1947–48, the present writer first offered at Harvard University a course of lectures entitled "Organization of Large-Scale Digital Calculating Machinery." This course included a treatment of general-purpose calculators from the functional and operational point of view and a discussion of individual components such as selection circuits, storage devices, adders, multipliers, etc. The circuits chosen for discussion were all taken from operating computers, and the case method of instruction was largely employed.

At the same time, the Staff of the Computation Laboratory was engaged in the design of computing and control circuits for the Mark III Calculator, making use of experimental results obtained from a research program extending over a ten-year period and the empirical methods of circuit synthesis at that time so widely employed. Thus, both in the laboratory and in the classroom, ample opportunity was offered to observe that the lack of adequate mathematical methods for the investigation of the functional behavior of electronic control circuits represented the largest single obstacle to the rapid development of the subject and to the instruction of students looking forward to an active career in the field.

Harry Rowe Mimno, Professor of Applied Physics, kindly attended the aforementioned course of lectures for two successive years. In addition to giving much valuable advice, Professor Mimno completely concurred with the opinion of the lecturer as to the desirability of undertaking a detailed investigation of the general subject of electronic-control-circuit synthesis. At this time a contract was effected between the United States Air Force and Harvard University which made it possible for the Staff of the Computation Laboratory to embark on a program of research in connection with electronic components for use in computing machinery. This contract, administered by Donald D. Foster for the Air Force, wisely stipulated that development of a general nature, rather than the construction of particular components, was to be the end in view, and thereby made available the opportunity to undertake the investigations leading to the results recorded in the present volume.

Work commenced in April 1948 and led to the sections on control-circuit theory included in the Progress Reports of the Computation Laboratory. For the results included in seven of these, William Burkhart, Theodore Kalin, and the present writer were jointly responsible.

In August 1949 Mr. Burkhart left the Computation Laboratory to join the research staff of the Monroe Calculating Machine Company. Mr. Kalin also left, in December 1949, and became associated with the Air Force Cambridge Research Laboratories. Thereafter the work was concluded with the collaboration of Peter Strong, who, together with the present writer, prepared the pertinent sections of Progress Reports Nos. 8, 13, and 16.

Although those already mentioned were primarily responsible for the production of this volume, many other members of the Staff of the Computation Laboratory have made important contributions. Martha Whitehouse constructed the minimizing charts used so profusely throughout this book, and in addition prepared minimizing charts of seven and eight variables for experimental purposes. J. Orten Gadd, Jr. designed the control tapes employed by the Automatic Sequence-Controlled Calculator in connection with the derivation of the Table of the Switching Functions of Four Variables contained in the Appendix. Frank Gucker and Robert Burns assisted in the construction of the Table of Input Rearrangements also included in the Appendix. Chapter X was greatly influenced by the work of An Wang concerning selenium-rectifier circuits. This chapter also includes mention of the methods of mounting small disk rectifiers developed by Robert E. Wilkins. The work of Benjamin Moore, Marshall Kincaid, Richard Hofheimer, Charles A. Coolidge, Jr., Way Dong Woo, Gerrit Blaauw, Michele Canepa of the Olivetti Company, Ltd., Ivrea, Italy, and others has been used to advantage throughout the book. Particular contributions have been made to Chapters XII and XIII by Dr. Wang and Mr. Coolidge. As previously mentioned, the sections on circuit theory in the Progress Reports represent the chief source of material for the present volume. Several of these reports, however, have undergone revision at the hands of Mr. Strong and the present writer.

As regards the mathematical approach to the subject matter of this volume it should be noted that several alternatives exist. The methods of the propositional calculus have been frequently suggested for use in this connection. Again, Boolean algebra was employed by Claude E. Shannon in his discussion of relay circuits. It is believed, however, that the algebraic approach adopted in the present volume provides a particularly convenient vehicle of thought and has the considerable advantage of lying within the province of the average reader's previous mathematical experience. Although this opinion has in part been confirmed by the experience of the Staff of the Computation Laboratory in the design of the Mark IV Calculator, final confirmation must wait until a reasonably large number of persons have had the opportunity to apply the methods of this volume. Hence, the present writer is obliged to record that the general algebraic approach, the switching function, the vacuum-tube operator, and the minimizing chart are his proposals, and that he is responsible for their inclusion herein.

In addition to the course of lectures at Harvard University, some of the material in the book has been presented in lecture form on other occasions. Mr. Kalin presented a brief summary of Chapters I to IV before the Association for Computing Machinery at Oak Ridge National Laboratory in April 1949. Mr. Burkhart gave a similar lecture before the Institute of Radio Engineers in May 1949 at Cambridge. Lectures have also been given on the subject by the present writer in the course of an European tour during January and February of 1951 at the International Colloquium "Les Machines à Calculer et la Pensée Humaine" at the Institut Blaise Pascal, Paris; at the Kunglig Tekniska Högskola, Stockholm, under the auspices of Matematikmaskinnämnden; the Chalmers Tekniska Högskola, Göteborg; the Eidgenössische Technische Hochschule, Zurich; and at the Institut für Praktische Mathematik, Technische Hochschule, Darmstadt. Professor Leon Brillouin, formerly of Harvard University and now with the International Business Machines Corporation, Professor Douglas Hartree of the University of Cambridge, Professor Charles Manneback of the University of Louvain, Professor Dr. Alwin Walther of the Institut für Praktische Mathematik, Darmstadt, and Dr. John Bowman of the Mellon Institute of Industrial Research have all discussed parts of the manuscript, and by their interest lent especial encouragement to those engaged in the task of compiling this volume.

The Staff of the Computation Laboratory wishes to express its appreciation of the kind coöperation of the Harvard University Press in making the publication of this and other volumes of the *Annals* possible; in particular Joseph D. Elder, Science Editor of the Press, has rendered valuable editorial assistance.

Jacquelin Sanborn prepared the manuscript for publication with the assistance of Carmela M. Ciampa, who drew the many figures of the book. The photographs of rectifier circuits and the Mark I multiply unit, as well as the films used in making the plates from which the book was printed, were prepared under the direction of Paul Donaldson, photographer of Cruft Laboratory, in part assisted by Robert Burns.

It is a pleasure to acknowledge the support given by contract W19-122-ac-24 between the United States Air Force and Harvard University, without which the present work could not have been undertaken.

HOWARD H. AIKEN

Cambridge, Massachusetts
Janaury 1951

Algebraic Approach

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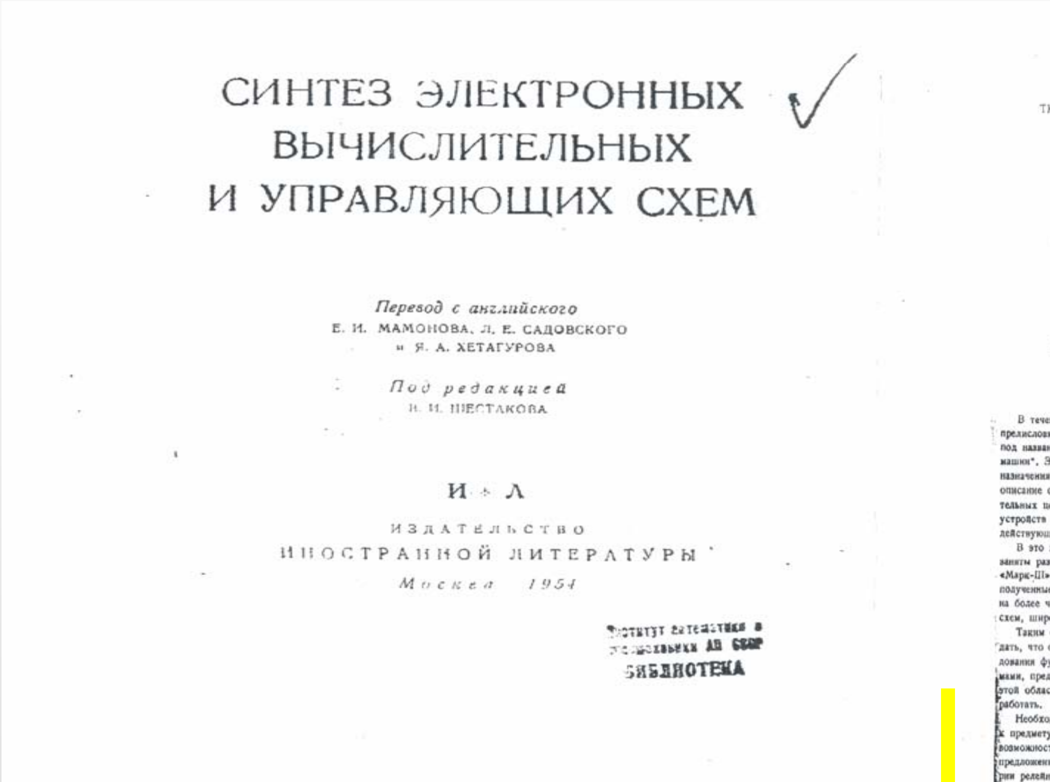
Howard H. Aiken, 1951

Shestakov and Translation into Russian



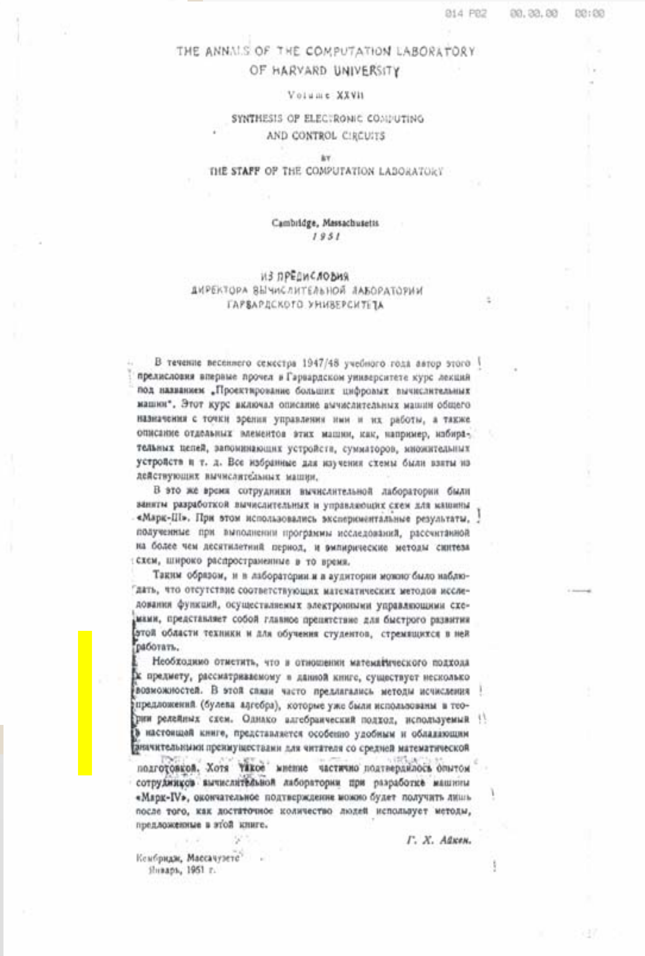
СИНТЕЗ ЭЛЕКТРОННЫХ
ВЫЧИСЛИТЕЛЬНЫХ
И УПРАВЛЯЮЩИХ СХЕМ

Перевод с английского
Е. И. МАМОНОВА, Л. Е. САДОВСКОГО
и Я. А. ХЕТАГУРОВА

Под редакцией
В. И. ШЕСТАКОВА

И Л

ИЗДАТЕЛЬСТВО
ИНОСТРАННОЙ ЛИТЕРАТУРЫ
Москва 1954



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THE ANNALS OF THE COMPUTATION LABORATORY
OF HARVARD UNIVERSITY
Volume XXVII

SYNTHESIS OF ELECTRONIC COMPUTING
AND CONTROL CIRCUITS
BY
THE STAFF OF THE COMPUTATION LABORATORY

Cambridge, Massachusetts
1951

ИЗ ПРЕДИСЛОВИЯ
ДИРЕКТОРА ВЫЧИСЛИТЕЛЬНОЙ ЛАБОРАТОРИИ
ГАРВАРДСКОГО УНИВЕРСИТЕТА

В течение весеннего семестра 1947/48 учебного года автор этого предисловия впервые прочел в Гарвардском университете курс лекций под названием „Проектирование больших цифровых вычислительных машин". Этот курс включал описание вычислительных машин общего назначения с точки зрения управления ими и их работы, а также описание отдельных элементов этих машин, как, например, избирательных цепей, запоминающих устройств, сумматоров, множительных устройств и т. д. Все избранные для изучения схемы были взяты из действующих вычислительных машин.

В это же время сотрудники вычислительной лаборатории были заняты разработкой вычислительных и управляющих схем для машины «Марк-III». При этом использовались экспериментальные результаты, полученные при выполнении программы исследований, рассчитанной на более чем десятилетний период, и эмпирические методы синтеза схем, широко распространенные в то время.

Таким образом, и в лаборатории и в аудитории можно было наблюдать, что отсутствие соответствующих математических методов исследования функций, осуществляемых электронными управляющими схемами, представляет собой главное препятствие для быстрого развития этой области техники и для обучения студентов, стремящихся в ней работать.

Необходимо отметить, что в отношении математического подхода к предмету, рассматриваемому в данной книге, существует несколько возможностей. В этой связи часто предлагались методы исчисления предложений (булева алгебра), которые уже были использованы в теории релейных схем. Однако алгебраический подход, используемый в настоящей книге, представляется особенно удобным и обладающим значительными преимуществами для читателя со средней математической подготовкой. Хотя такое мнение частично подтвердилось опытом сотрудников вычислительной лаборатории при разработке машины «Марк-IV», окончательное подтверждение можно будет получить лишь после того, как достаточное количество людей использует методы, предложенные в этой книге.

Г. Х. Айкен.

Кембридж, Массачузетс
Январь, 1951 г.

Switching Theory and DSP

Different interpretation of existing methods for
better understanding and improved exploiting in practice

A unified approach to various results,
their extensions, and generalizations

Derivation of completely new resutls for switching functions

Spectral Methods

Classical approaches

Fixed domain group, selected transforms

Change of basis functions

Preserving some but not all useful properties

Mostly FFT-like algorithms

Reduced number of non-zero coefficients

Disadvantage - missing of some properties

Group-theoretic approach

Fixed transform (Fourier), selected domain groups

Basic Characteristics of Future Computing Technologies

Regularity

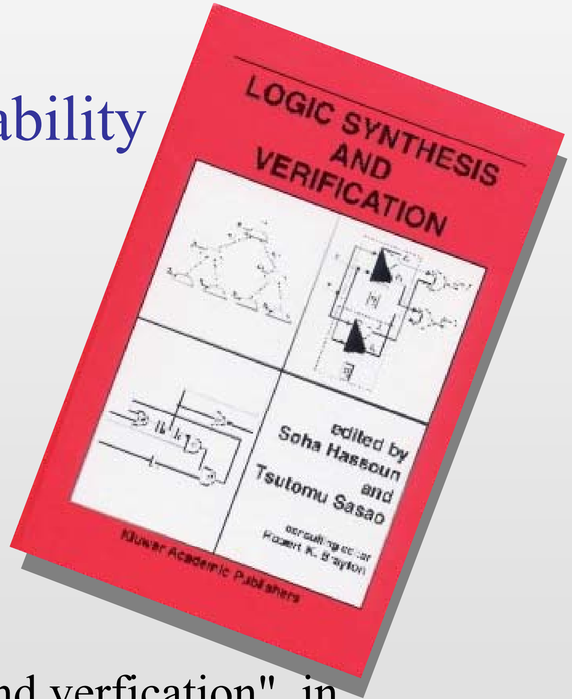
Programmability and re-programmability

Delay constrains

Deep sub-micron effects

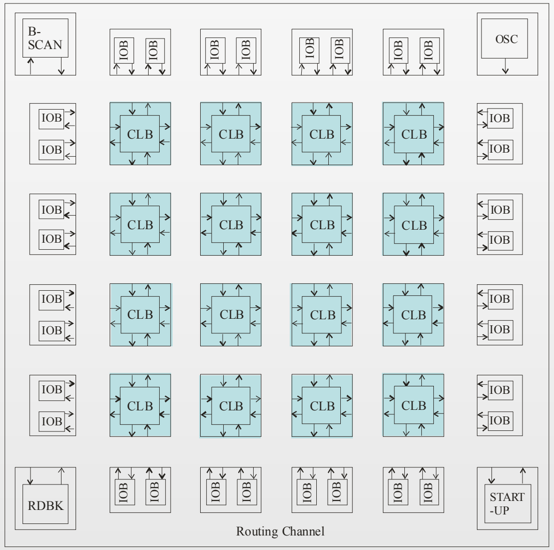
Logic span

Reusability



Brayton, R.K., "The future of logic synthesis and verfication", in Hassoun, S., Sasao, T., (eds.), *Logic Synthesis and Verication*, Kluwer Academic Publishers, Boston, MA, USA, 2002, 403-434.

Some Available FPGA



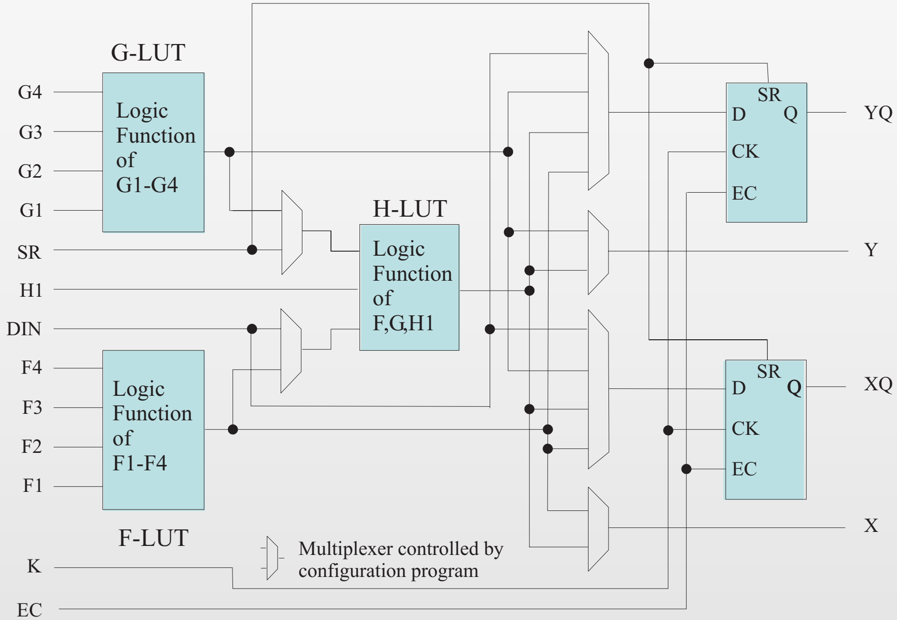
CLB - functional elements to implement logic

IOB - interface between the package pins and internal signal lines

Routing Channels - pats to interconnect the inputs and outputs of CLB and IOBs

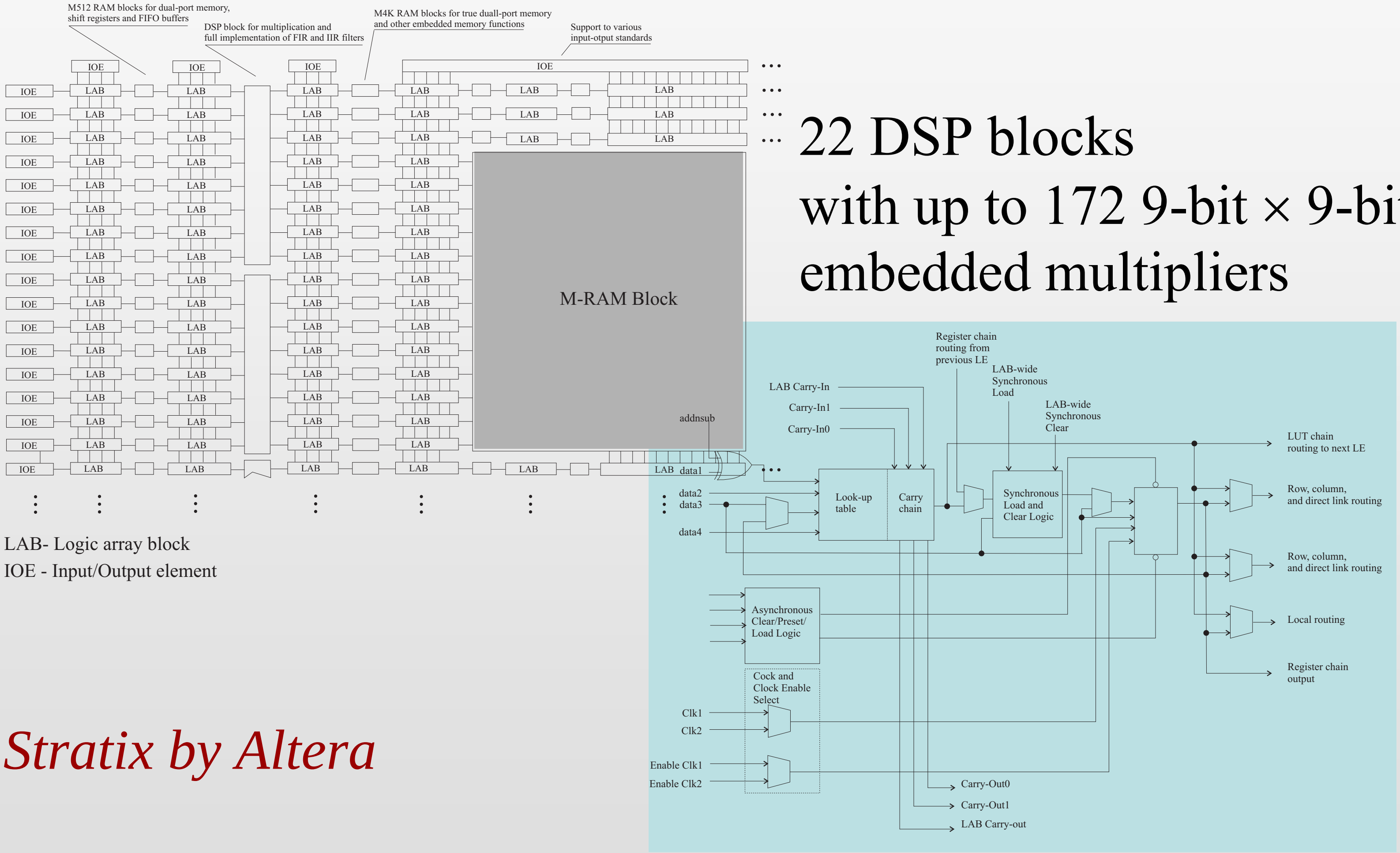
RDBK - read back the content of the configuration memory and the level of certain internal nodes

START-UP - start-up bytes of data to provide four clocks for the start-up sequence at the end of configuration



Spartan by Xilinix

FPGA with DSP Block



LAB- Logic array block
IOE - Input/Output element

22 DSP blocks
with up to 172 9-bit × 9-bit embedded multipliers

Stratix by Altera

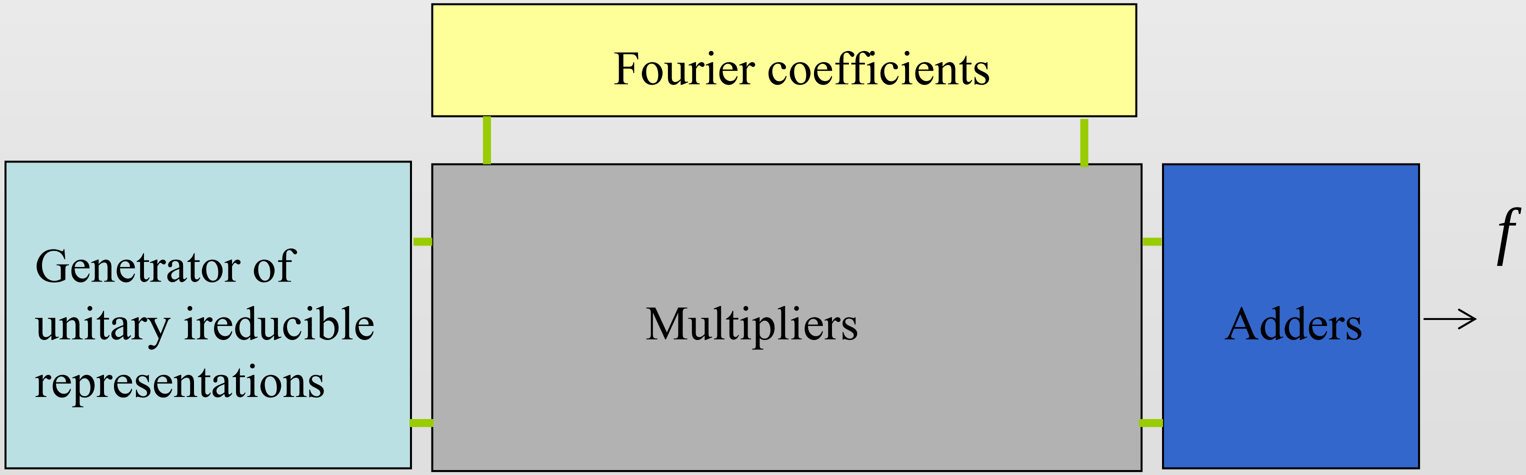
Design from Fourier Representations

Design principle

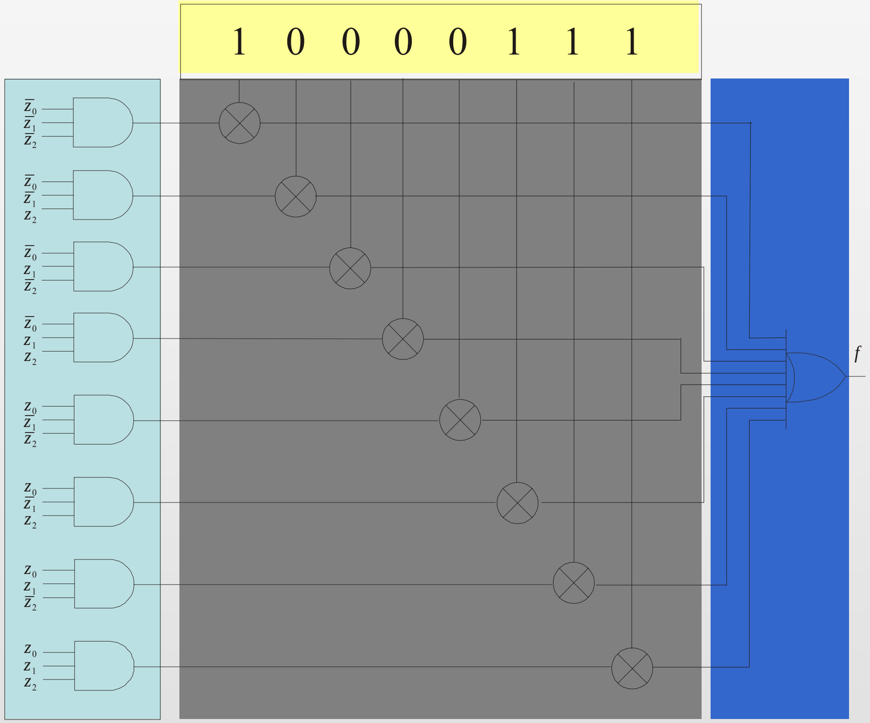
Decomposition of f in terms of Fourier coefficients

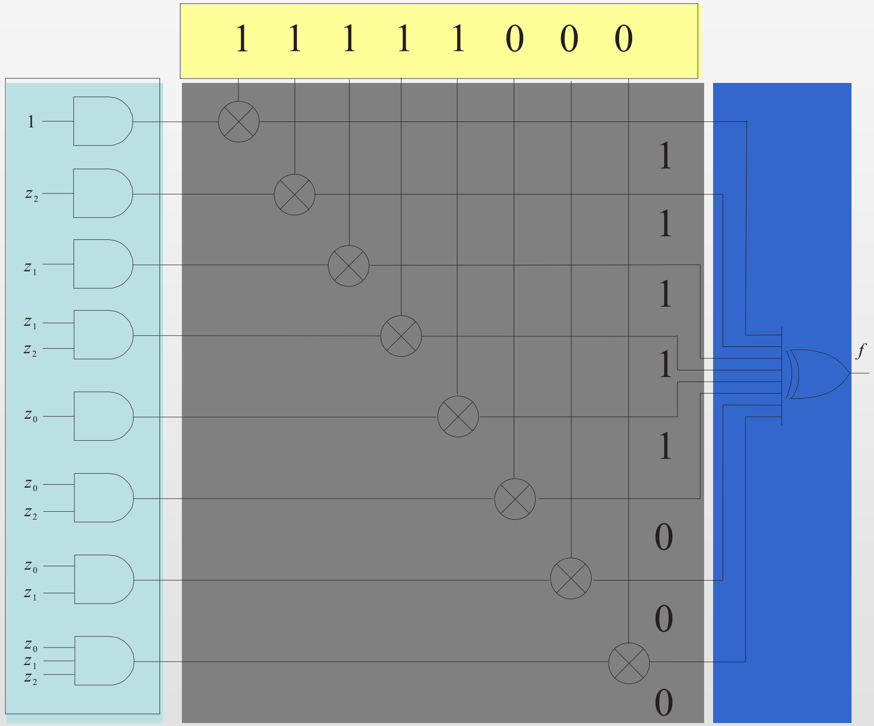
Realization of coefficients

Network of subnetworks for the coefficients



SOP and Reed-Muller Realizations

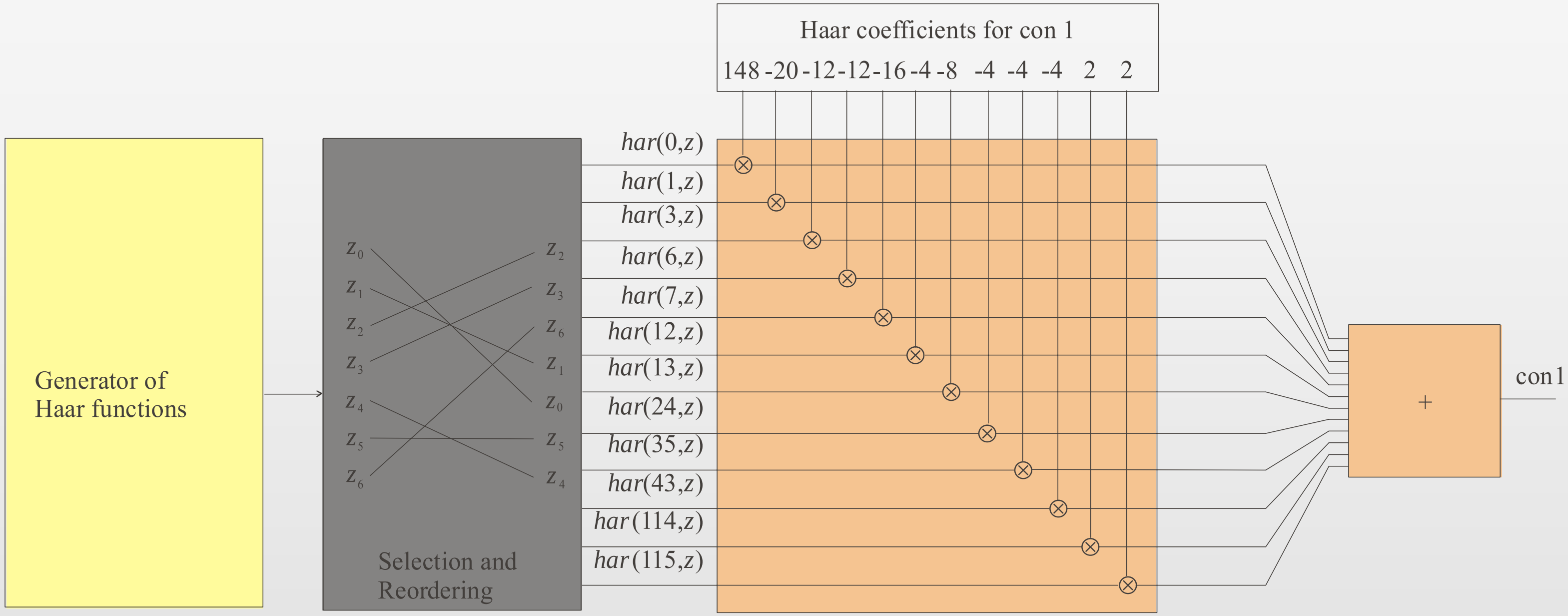




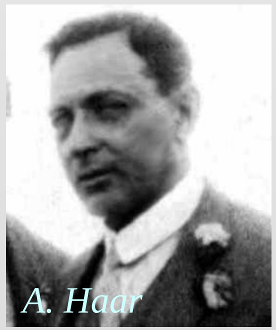
Transform $\mathbf{I}(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mathbf{R}(1) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Circuit from Haar Series



Transform – $(2^7 \times 2^7)$ Haar transform



Fourier Transform

Fourier $G = R$

$$f(x) = \int_{-\infty}^{\infty} S_f(w) e^{2\pi wx} dw$$

$$S_f(w) = \int_{-\infty}^{\infty} f(x) e^{-2\pi iwx} dx$$

Spectral Transforms

|G| - finite

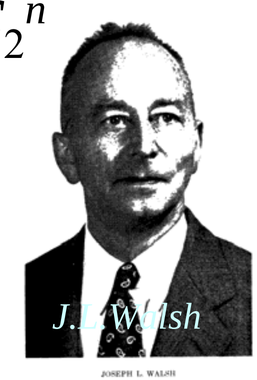
$$f(x) = \sum_{w=0}^{g-1} S_f(w)\chi(w,x)$$

$$S_f(w) = g^{-1}\sum_{x=0}^{g-1} f(x)\chi^*(w,x)$$

Walsh = Fourier on C_2^n

$$f = \sum_{w} S_f(w) wal(w,x)$$

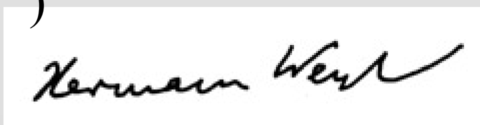
$$S_f = 2^{-n}\sum_{x} f(x) wal(w,x)$$



Fourier on G = Finite non-Abelian

$$\mathbf{f}(x) = (f^{(i,j)}(x)) = \sum_{w=0}^{K-1} Tr(\mathbf{S}_f^{(i,j)}(w)\mathbf{R}_w(x))$$

$$\mathbf{S}_f(w) = (S_f^{(i,j)}(w)) = r_w g^{-1}\sum_{u=0}^{g-1} f^{(i,j)}(u)\mathbf{R}_u(u^{-1})$$



Quaternion Group

$$\mathbf{I}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\quad \mathbf{A}=\begin{bmatrix}1&0\\0&-1\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}-1&0\\0&1\end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\quad \mathbf{D}=\begin{bmatrix}0&1\\1&0\end{bmatrix}\quad \mathbf{E}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$$

Group representations

$r_w = 1, r_w = 2$

Coefficients

complex numbers

and (2×2) matrices

Q_2

x	$\mathbf{R}_0$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$
0	1	1	1	1	**I**
1	1	1	-1	-1	*i***A**
2	1	1	1	1	-**I**
3	1	1	-1	-1	*i***B**
4	1	-1	1	-1	**C**
5	1	-1	-1	1	-*i***D**
6	1	-1	1	-1	**E**
7	1	-1	-1	1	*i***D**
	$r_0 = 1$	$r_1 = 1$	$r_2 = 1$	$r_3 = 1$	$r_4 = 2$

Fourier Transform

Transform matrix

$$[\mathbf{Q}]^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ \mathbf{I} & 2i\mathbf{B} & -2\mathbf{I} & 2i\mathbf{A} & 2\mathbf{E} & 2i\mathbf{D} & 2\mathbf{C} & -2i\mathbf{D} \end{bmatrix}$$

Transform pair

$$[\mathbf{S}_f] = [\mathbf{Q}_2]^{-1}[\mathbf{F}]$$

$$(1\times 5) = (5\times 8)(8\times 1)$$

Spectrum

$$[\mathbf{S}_f] = [\mathbf{S}_f(0) \quad \mathbf{S}_f(1) \quad \mathbf{S}_f(2) \quad \mathbf{S}_f(3) \quad \mathbf{S}_f(4)]^T$$

5

Function

$$[\mathbf{F}_m] = [\mathbf{f}(0) \quad \mathbf{f}(1) \quad \mathbf{f}(2) \quad \mathbf{f}(3) \quad \mathbf{f}(4) \quad \mathbf{f}(5) \quad \mathbf{f}(6) \quad \mathbf{f}(7)]^T$$

8

Fourier Tranform Matrix

$$\mathbf{Q}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 & i & 0 & 0 & -i \\ 1 & 1 & 1 & 1 & -1 & 0 & 0 & -1 \\ 1 & 1 & -1 & -1 & -i & 0 & 0 & i \\ 1 & -1 & 1 & -1 & 0 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & -i & -i & 0 \\ 1 & -1 & 1 & -1 & 0 & 1 & -1 & 0 \\ 1 & -1 & -1 & 1 & 0 & i & i & 0 \end{bmatrix}$$

$$\mathbf{F} = \left[f(0) \quad f(1) \quad f(2) \quad f(3) \quad f(4) \quad f(5) \quad f(6) \quad f(7) \right]^T$$

$$\mathbf{S}_f = \left[S_f(0) \quad S_f(1) \quad S_f(2) \quad S_f(3) \quad S_f(4) \quad S_f(5) \quad S_f(6) \quad S_f(7) \right]^T$$

8

$$\mathbf{Q}_2^{-1} = \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 2 & -2i & -2 & 2i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 2i & 2 & -2i \\ 0 & 0 & 0 & 0 & 2 & 2i & -2 & -2i \\ 2 & 2i & -2 & -2i & 0 & 0 & 0 & 0 \end{bmatrix}$$

No restrictions to entiries in **F**

Could be matrices

Fourier Representations on Finite Groups

Groups of the same order, subgroups of different orders

$$G = \mathop{\times}_{i=1}^{n} G_i \qquad g = \prod_{i=0}^{n} g_i$$

The optimization method = Selecting suitable groups

$C_2^7, C_2C_4^3, C_2C_8^2, C_2^4q_2^2, C_4^2q_2$ $\quad n = 7$

$C_2^8, C_4^4, C_2^5q_2, C_2^5C_8, C_2^2q_2^2, C_4q_2^2$ $\quad n = 8$

$C_2^9, C_2C_4^4, C_2^6q_2, C_2^6C_8, C_8^3, q_2^3$ $\quad n = 9$

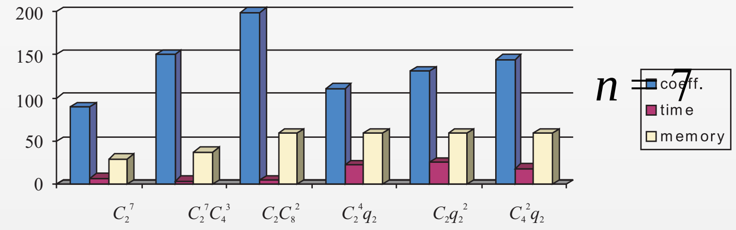
$C_2^{10}, C_4^5, C_2^7C^8, C_2^4C_8^2, C_2C_8^3, C_2^7q_2, C_2^4q_2^2, C_2q_2^3$ $\quad n = 10$

Complexity of Fourier Representations

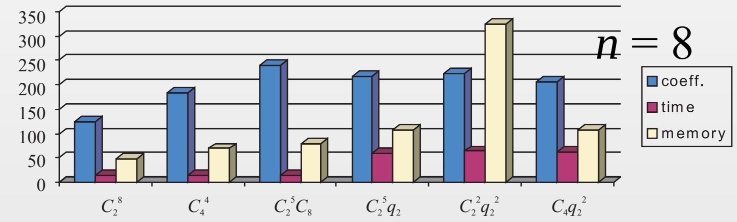
of non-zero coefficients
Calculation time
Memory

f	In	Coff.	Time	Memory
5xp1	7	$C_2 q_2^2$	$C_2 C_8^2$	C_2^7
rd84	8	C_4^4	$C_2^5 C_8$	C_2^8
clip	9	C_2^9	q_2^3	C_2^9
add5	10	C_2^{10}	$C_2^4 C_8^2$	C_2^{10}
mul5	10	C_2^{10}	C_4^5	C_2^{10}
sao2	10	$C_2^7 q_2$	$C_2 C_8^3$	C_2^{10}
ex1010	10	C_2^{10}	C_4^5	C_2^{10}

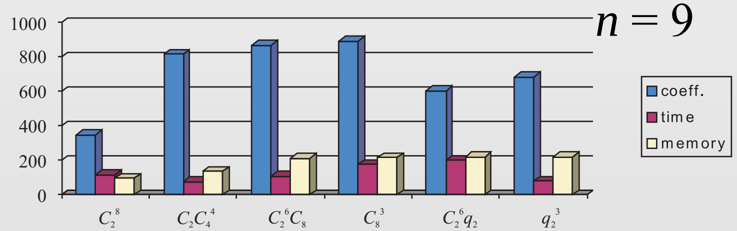
$n = 7$



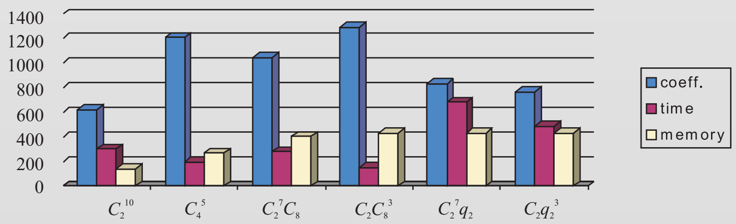
$n = 8$



$n = 9$



$n = 10$



Coefficients and Bits

	C_2^7		$C_2C_4^3$		$C_2C_8^2$		$C_2^4\, q_2$		$C_2q_2^2$		$C_2^4\, q_2$	
f	coeff.	bits	coeff.	bits	coeff.	bits	coeff.	bits	coeff.	bits	coeff.	bits
cos(x)	124	1099	240	1592	248	1647	182	1280	189	722	222	1477
exp(x)	128	1472	224	1904	240	2152	192	562	144	562	192	1477
ln(x)	128	1485	240	2087	248	2211	192	577	220	577	220	577
sin(x)	128	862	226	1146	246	1438	184	953	210	715	228	1185
tan(x)	128	1421	238	2102	248	2156	192	548	212	548	236	548
x^2	103	752	210	997	231	1207	159	851	189	683	189	1007
x^3	128	859	220	1148	246	530	168	984	210	701	234	1177
x^4	111	869	202	1230	248	1612	167	1053	102	709	231	1308
$x5$	115	931	236	1360	246	1686	178	1143	202	731	232	1396
av.	121	1083	226	1506	195	1626	168	883	186	660	220	1128

Bits and 1-Bits

	C_2^7		$C_2C_4^3$		$C_2C_8^2$		$C_2^4\,q_2$		$C_2q_2^2$		$C_2^4\,q_2$	
f	bits	1-bits	bits	1-bits	bits	1-bits	bits	1-bits	bits	1-bits	bits	1-bits
cos(x)	1099	360	1592	713	1647	788	1280	496	722	126	1477	646
exp(x)	1472	608	1904	888	2152	992	562	35	562	35	1477	35
ln(x)	1485	656	2087	969	2211	1135	577	35	577	35	577	35
sin(x)	862	336	1146	558	1438	726	953	453	715	169	1185	610
tan(x)	1421	501	2102	1004	2156	1021	548	20	548	20	548	20
x^2	752	157	997	394	1207	506	851	274	683	110	1007	359
x^3	859	323	1148	538	530	17	984	409	701	136	1177	582
x^4	869	247	1230	464	1612	770	1053	383	709	127	1308	626
$x5$	931	262	1360	641	1686	799	1143	392	731	125	1396	617
av.	1083	383	1506	685	1626	750	883	277	660	98	1128	392

Coefficients, Time, Memory

add5	11	26	21	31	41	18	25	32
mul5	36	102	96	262	229	78	112	152
sao2	1024	1816	1464	2016	1956	960	1830	1540
ex1010	1024	2016	1892	1968	2032	1536	1792	1920
fun10	826	1823	1638	1984	1980	1343	1791	1740
av.	584.2	1156.6	1022.2	1252.2	1247.0	787.0	1110.0	1076.8
add5	101	188	211	80	162	792	331	762
mul5	103	92	113	130	129	502	280	538
sao2	414	201	280	210	157	630	370	528
ex1010	431	244	317	280	170	617	491	403
fun10	457	275	486	251	243	874	410	695
av.	301	200	281	190	172	683	376	585
add5	142	269	433	388	433	428	380	428
mul5	184	266	386	386	430	426	379	426
sao2	187	269	282	377	433	421	367	421
ex1010	187	269	433	385	433	438	388	438
fun10	187	269	433	388	433	438	385	438
av.	177	268	402	458	432	430	380	430

Design Recommendations

C_2^n	usually (not always) requires smallest number of non-zero coefficients 5xp1, rd84, sao2 → $C_2 q_2^2$, C_4^4, C_2^7
$C_2^i q_2^r$	requires smallest number of bits and 1 bits
$C_2^i C_8^r$, C_4^i	fastes computations
C_2^n	smallest memory

Closing Remarks

Relating of DSP and Switching theory assumes cahange of the underlying algebraic structures usually used in study of switching functions

Due to that

Different intepretations of existing methods and techniques

A unified way for extensions and genralizations of theory

Derivation of new results in Switching Thory by borrowing ideas from DSP

Acknowledgment

TICSP
Tampere International Center for Signal Processing