Minimization of Multi-Valued Functions in Decomposition of a System of Completely Specified Boolean Functions

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The problem: Given a system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ where

$\boldsymbol{y}=(y_1, y_2, \ldots, y_m)$, $\boldsymbol{x}=(x_1, x_2, \ldots, x_n)$,

$\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}), \ldots,\ f_m(\boldsymbol{x}))$,

and a partition of set of arguments $X=\{x_1, x_2, \ldots, x_n\}$ into subsets Q, W and E,

a superposition $\boldsymbol{y}=\boldsymbol{g}(u, \boldsymbol{c})$, $u=h(\boldsymbol{d})$ where the components of $\boldsymbol{c}$ are the variables from $E \cup W$ and the components of $\boldsymbol{d}$ from $Q \cup E$ should be found with the following restrictions:

$e < | Q \cup E |$, $e + | W \cup E | < n$,

where e is the minimum number of bits encoding the values of u.

System of functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ is given by matrices:
U is a ternary matrix whose rows represent terms of DNFs of the functions,
V is a binary matrix whose each column shows which terms belongs to DNF of a certain function.

$$U = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & - & 0 \\ 1 & 0 & - & - & 1 & - \\ 0 & 1 & 0 & 1 & 0 & 1 \\ - & 0 & 1 & 0 & - & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \qquad V = \begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$Q = \{x_3, x_6\}$, $W = \{x_4, x_5\}$, $E = \{x_1, x_2\}$.
$\boldsymbol{d} = (x_1, x_2, x_3, x_6)$, $\boldsymbol{c} = (x_1, x_2, x_4, x_5)$.

To obtain $\boldsymbol{y}=\boldsymbol{g}(u, \boldsymbol{c})$, $u=h(\boldsymbol{d})$ the technique of ternary matrix cover is used.

The family π of subsets of the set of rows of a ternary matrix U with n columns is called cover of U if for every binary vector $\boldsymbol{b}$ of length n there is a subset in π containing all rows of U (ternary vectors) that absorb $\boldsymbol{b}$.

Matrix U is divided into matrices

$$U^{1} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & - & - \\ 0 & 1 & 0 & 1 \\ - & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \qquad U^{2} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & - \\ 1 & 0 & - & 1 \\ 0 & 1 & 1 & 0 \\ - & 0 & 0 & - \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

The blocks of cover π^1 of U^1 and corresponding functions are

$\pi^1{}_1 = \{1\}$, $\quad \pi^1{}_1(\boldsymbol{d}) = x_1 x_2 \overline{x}_3 \overline{x}_6$;

$\pi^1{}_2 = \{2\}$, $\quad \pi^1{}_2(\boldsymbol{d}) = x_1 \overline{x}_2 \overline{x}_3 \vee x_1 \overline{x}_2 \overline{x}_6$;

$\pi^1{}_3 = \{3\}$, $\quad \pi^1{}_3(\boldsymbol{d}) = \overline{x}_1 x_2 \overline{x}_3 x_6$;

$\pi^1{}_4 = \{4\}$, $\quad \pi^1{}_4(\boldsymbol{d}) = \overline{x}_1 \overline{x}_2 x_3 x_6$;

$\pi^1{}_5 = \{2,4\}$, $\quad \pi^1{}_5(\boldsymbol{d}) = x_1 \overline{x}_2 x_3 x_6$;

$\pi^1{}_6 = \varnothing$, $\quad \pi^1{}_6(\boldsymbol{d}) = x_2 x_3 \vee \overline{x}_1 \overline{x}_2 \overline{x}_3 \vee \overline{x}_1 \overline{x}_6 \vee x_1 x_2 x_6$.

Multi-valued function $u = h(\boldsymbol{d})$ can be given by the following matrices:

$$\Psi = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & - \\ 1 & 0 & - & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \qquad \Omega = \begin{array}{c} u \\ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

The predicate system $\boldsymbol{y}=\boldsymbol{g}(u, \boldsymbol{c})$ is incompletely specified and can be given by matrices

$$
Z' = \begin{array}{c} \begin{matrix} u & & & & & & & x_1 & x_2 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 0&1&0&0&0&0&.&1&1&1&-\\ 0&0&1&0&0&1&.&1&0&-&1\\ 0&0&0&0&0&1&.&1&0&0&-\\ 0&0&0&1&0&0&.&0&1&1&0\\ 0&0&0&0&1&0&.&0&0&0&- \end{bmatrix} \end{array} \begin{matrix} 1\\2\\3\\4\\5 \end{matrix}
\qquad
V' = \begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{bmatrix} 0&0&1\\ 1&1&1\\ 1&1&0\\ 0&1&0\\ 1&1&0 \end{bmatrix} \end{array} \begin{matrix} 1\\2\\3\\4\\5 \end{matrix}
$$

and matrix

$$
F'' = \begin{array}{c} \begin{matrix} u & & & & & & & x_1 & x_2 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 0&1&1&0&0&1&.&0&-&-&-\\ 0&1&0&1&0&0&.&-&0&-&-\\ 0&0&1&0&1&1&.&-&1&-&-\\ 0&0&0&1&1&0&.&1&-&-&-\\ 1&0&0&0&0&0&.&1&0&-&- \end{bmatrix} \end{array}
$$

giving the domain of "don't care".

The result of minimization by Espresso-MV program:

$$Z^M = \begin{array}{c} \begin{matrix} & u & & & & & & x_1 & x_2 & x_4 & x_5 \end{matrix} \\ \left[\begin{matrix} 0&0&0&1&0&0&.&-&-&1&0 \\ 0&1&0&0&0&0&.&-&-&1&- \\ 0&0&0&0&1&1&.&-&-&0&- \\ 1&1&1&1&1&1&.&1&0&-&1 \end{matrix}\right] \end{array} \begin{matrix} 1\\2\\3\\4 \end{matrix} \qquad V^M = \begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \left[\begin{matrix} 0&1&0 \\ 0&0&1 \\ 1&1&0 \\ 1&1&1 \end{matrix}\right] \end{array} \begin{matrix} 1\\2\\3\\4 \end{matrix}$$

Variable u can be replaced by variable $u^{\sim}$ that has the less set of values and, as a function, is given by matrices

$$\Psi = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_6 \end{matrix} \\ \left[\begin{matrix} 1&1&0&0 \\ 1&0&0&- \\ 1&0&-&0 \\ 0&1&0&1 \\ 0&0&1&1 \\ 1&0&1&1 \end{matrix}\right] \end{array} \begin{matrix} 1\\2\\3\\4\\5\\6 \end{matrix} \qquad \Omega^M = \begin{array}{c} u \\ \left[\begin{matrix} 1\\0\\0\\2\\3\\3 \end{matrix}\right] \end{array} \begin{matrix} 1\\2\\3\\4\\5\\6 \end{matrix}$$

After simplification the looked for superposition is given by the following matrices:

$$\Psi = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ - & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \qquad \Omega^M = \begin{array}{c} u \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$G^M = \begin{array}{c} \begin{matrix} u & & & & & x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & . & - & - & 1 & 0 \\ 0 & 1 & 0 & 0 & . & - & - & 1 & - \\ 0 & 0 & 0 & 1 & . & - & - & 0 & - \\ 1 & 1 & 1 & 1 & . & 1 & 0 & - & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \qquad H^M = \begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

Experimental results

<table>
<tr><th rowspan="2">Name</th><th rowspan="2">n</th><th rowspan="2">m</th><th rowspan="2">l</th><th rowspan="2">$|Q|$</th><th rowspan="2">$|W \cup E|$</th><th rowspan="2">$|E|$</th><th colspan="2">e</th></tr>
<tr><th>Method 1</th><th>Methods 2 and 3</th></tr>
<tr><td rowspan="4">Newtpla1.pla</td><td rowspan="4">10</td><td rowspan="4">2</td><td rowspan="4">4</td><td rowspan="4">6</td><td>4</td><td>0</td><td>3</td><td rowspan="4">3</td></tr>
<tr><td>5</td><td>1</td><td rowspan="2">2</td></tr>
<tr><td>6</td><td>2</td></tr>
<tr><td>7</td><td>3</td><td>1</td></tr>
<tr><td rowspan="4">Newtpla2.pla</td><td rowspan="4">10</td><td rowspan="4">4</td><td rowspan="4">9</td><td rowspan="4">6</td><td>4</td><td>0</td><td rowspan="3">3</td><td rowspan="4">3</td></tr>
<tr><td>5</td><td>1</td></tr>
<tr><td>6</td><td>2</td></tr>
<tr><td>7</td><td>3</td><td>2</td></tr>
<tr><td rowspan="4">alu1</td><td rowspan="4">12</td><td rowspan="4">8</td><td rowspan="4">19</td><td rowspan="4">9</td><td>3</td><td>0</td><td>9</td><td rowspan="2">9</td></tr>
<tr><td>4</td><td>1</td><td>8</td></tr>
<tr><td>5</td><td>2</td><td>7</td><td rowspan="2">8</td></tr>
<tr><td>6</td><td>3</td><td>6</td></tr>
<tr><td rowspan="4">T3</td><td rowspan="4">12</td><td rowspan="4">8</td><td rowspan="4">152</td><td rowspan="4">7</td><td>5</td><td>0</td><td>4</td><td rowspan="4">4</td></tr>
<tr><td>6</td><td>1</td><td>3</td></tr>
<tr><td>7</td><td>2</td><td rowspan="2">2</td></tr>
<tr><td>8</td><td>3</td></tr>
</table>

B12.pla	15	9	431	8	7	0	6	6
					8	1	5	
					9	2		
					10	3	4	
In0.pla	15	11	138	9	6	0	6	6
					7	1	5	
					8	2		
					9	3	4	
T481	16	1	481	9	7	0	3	3
					8	1		
					9	2		
					10	3		
Ex7	16	5	123	9	7	0	8	8
					8	1	7	
					9	2	6	
					10	3		
Cordic.pla	23	2	1206	12	11	0	4	4
					12	1		
					13	2		
					14	3		