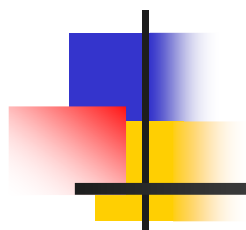


Relation between Boolean functions and spreading sequences for MC-CDMA transmission



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Outline

- Multicarrier CDMA (MC-CDMA) and a major problem with it
- Haar wavelet packets
- Binary-valued spreading transforms and Boolean logic
- Relation to the bent functions
- Chrestenson transform and complex-valued counterparts of Haar wavelet packets
- Multiple-valued spreading transforms and multiple-valued logic



What is MC-CDMA(1)

- MC-CDMA is a radio access technique, which will potentially play role in 4G mobile telephony
- MC-CDMA = OFDM + CDMA
 - OFDM - Orthogonal Frequency Division Multiplexing
 - CDMA - Code Division Multiple Access

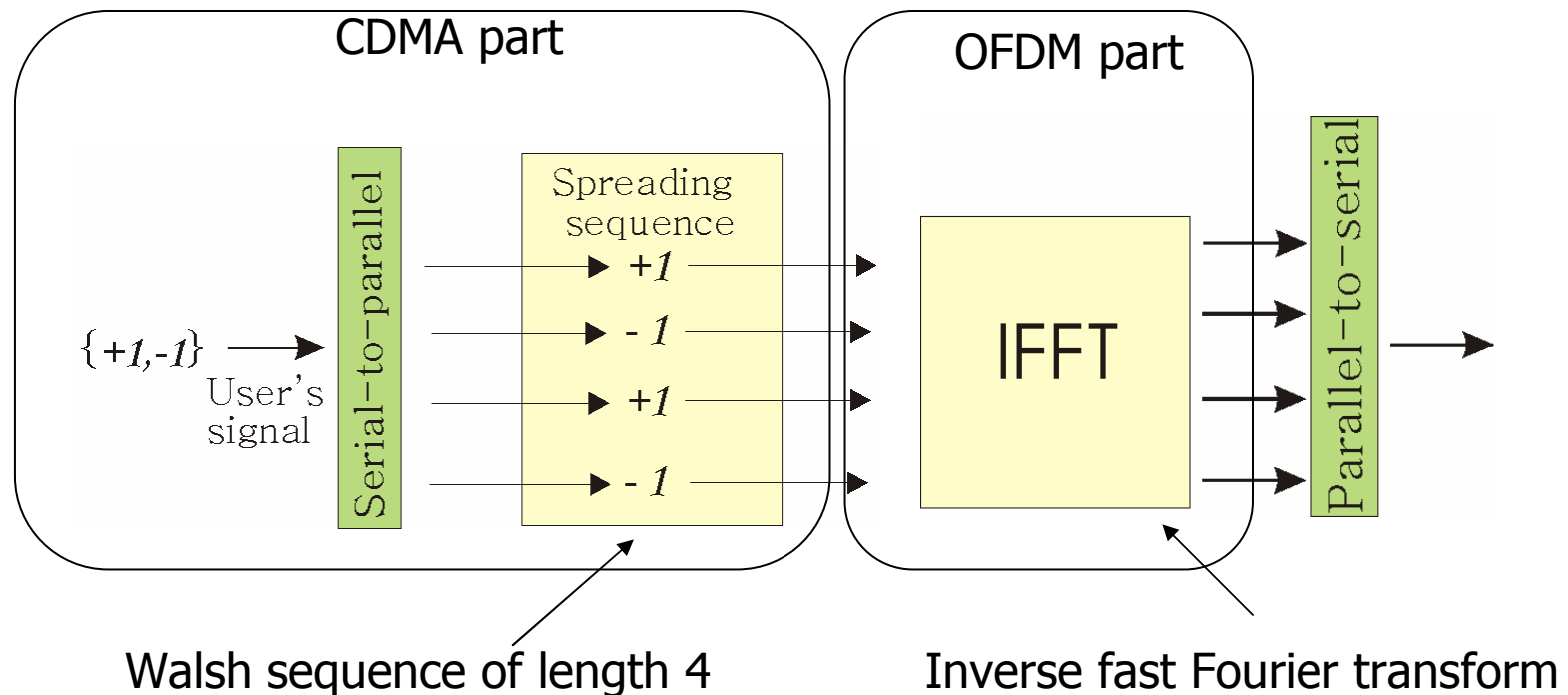


What is MC-CDMA(2)


3G
CDMA + OFDM = 4G
MC-CDMA

What is MC-CDMA(2)

In MC-CDMA systems, the symbol is multiplied (spread) by user specific spreading sequence, and converted into a parallel data stream, which is then transmitted over multiple carriers



Major problem – peak-to-average power ratio (PAPR)


$$PAPR(s(t)) = \frac{\max_t |s(t)|^2}{\frac{1}{T_s} \int_0^{T_s} |s(t)|^2 dt},$$

period of signal $s(t)$

$$s(t) = \text{Re}\left\{ \sum_{k=1}^N c_k^{(l)} \cdot e^{j\omega_k} \right\}.$$

signal at the transmitter output

(k, l) -th element of the spreading matrix

element of the Fourier matrix

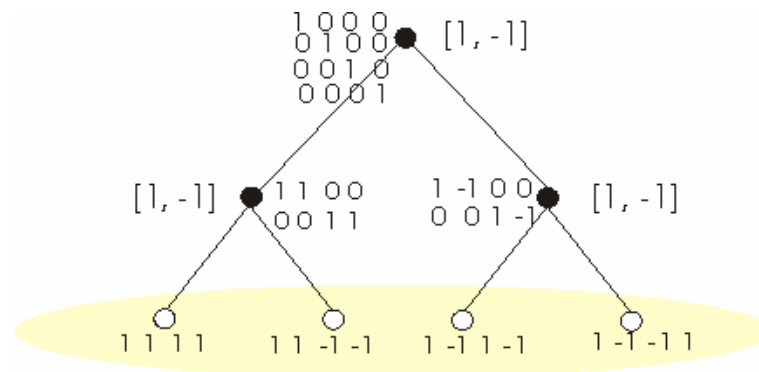
Problem: how to minimize PAPR of the resulting transmitted signal

Haar wavelet packets(1)

Walsh-Hadamard (WH) orthogonal matrix:

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, H_N = \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix}$$

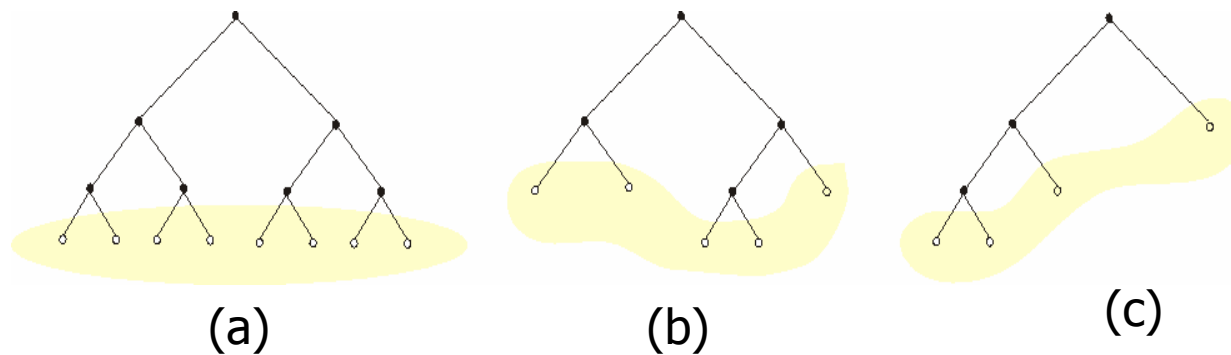
We can represent the way of constructing this matrix in a binary tree:



At the terminal nodes we have the rows of WH matrix

Haar wavelet packets(2)

We can take any pruned subtree of such a tree and combine the rows, corresponding to the terminal nodes of the resulting tree.



As a result, we will have a new $(N \times N)$ orthogonal matrix, where
Special case – Haar matrix – corresponds to Figure (c).

$$N = 2^n$$

New orthogonal spreading transforms

Consider the following recursive diagonal matrix:

$$D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_N = \begin{bmatrix} D_{\frac{N}{2}} & \circ \\ \circ & D_{\frac{N}{2}} \cdot \begin{bmatrix} I_{\frac{N}{4}} & \circ \\ \circ & -I_{\frac{N}{4}} \end{bmatrix} \end{bmatrix}$$

Let H_N be any $(N \times N)$ Haar wavelet packet (HWP) decomposition orthogonal matrix, and let us define G_N as

$$G_N = H_N \cdot D_N$$

Spreading with the new orthogonal matrix G_N results in a much lower PAPR of the resulting transmitted signal, than spreading with any HWP matrix H_N .

Boolean functions and spreading matrices(1)

Let $f_1(x), f_2(x), \dots, f_n(x)$ be all Boolean functions of $n = \log_2(N)$ variables of the form: $f_i(x) = x_i, i = 1, 2, \dots, n$.

The corresponding truth vectors are the first order Reed-Muller codes.

Example: $N=8$. In $\{+1, -1\}$ – encoding the truth vectors are

$$\vec{f}_1 = [1 \ 1 \ 1 \ 1 \ -1 \ -1 \ -1 \ -1]$$

$$\vec{f}_2 = [1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1] \quad \vec{f}_3 = [1 \ -1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1]$$

By $\tilde{D}_N^{(i)}, i = 1, 2, \dots, n$ we denote the following diagonal matrices:

$$\tilde{D}_N^{(i)}(k, k) = D_N^{(i)}(k, k) \cdot \vec{f}_i(k), \quad i, k = 1, 2, \dots, n$$

Boolean functions and spreading matrices(2)

The set of n new spreading matrices $\tilde{G}_N^{(i)}$, $i = 1, 2, \dots, n$ defined by

$$\tilde{G}_N^{(i)} = H_N \cdot \tilde{D}_N^{(i)}$$

result in an equally low PAPR, as G_N .

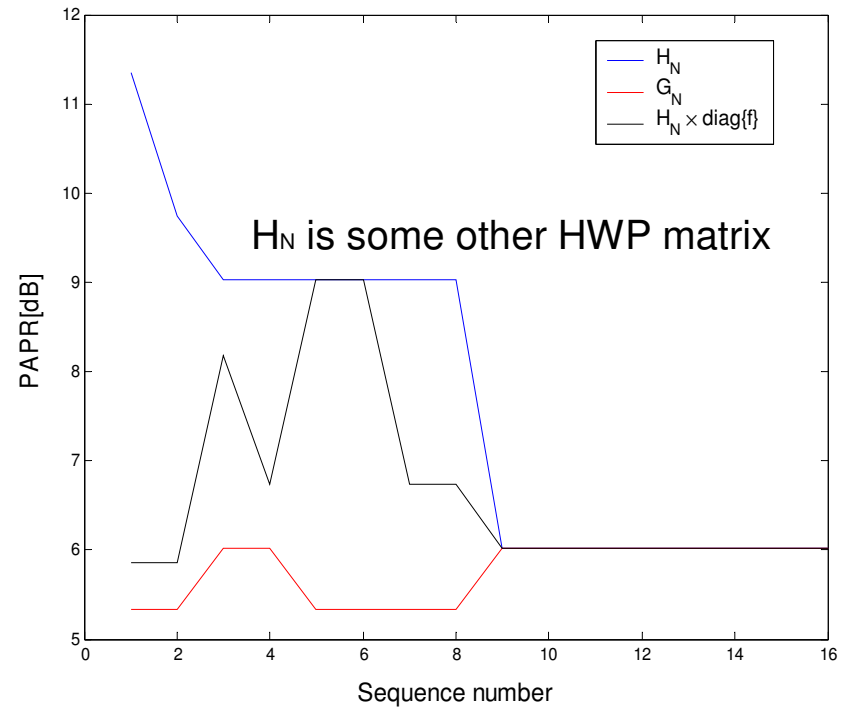
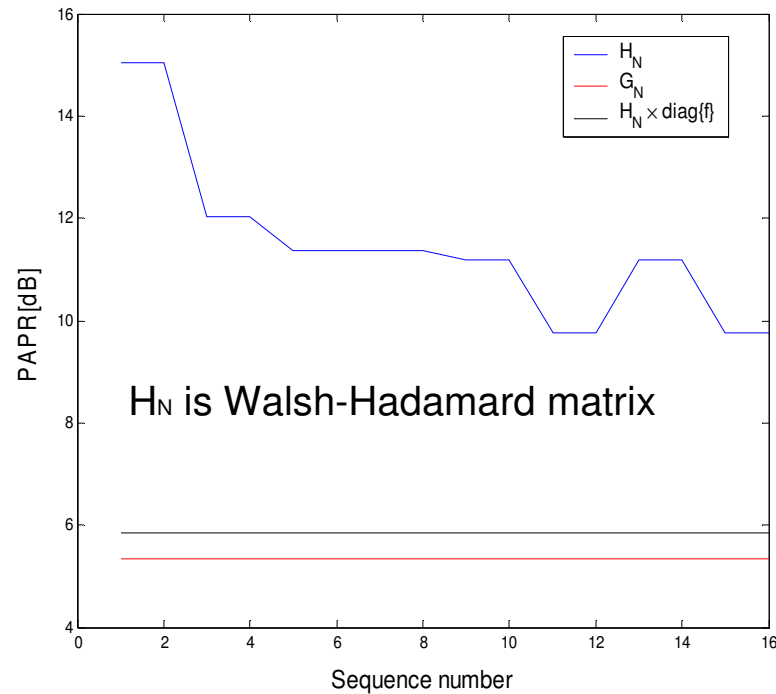
- Element-by-element multiplying the main diagonal of matrix D_N with the truth vector of a BF of the form in $\{+1, -1\}$ – encoding, we will get a new diagonal matrix, which retains the feature of reducing PAPR.
- In this way, for each HWP matrix H_N , we obtain $n+1$ new sets of orthogonal spreading sequences, which differ from H_N only by the signs of its columns, but have significantly better PAPR properties.
- Note: In the particular case, when H_N is the Walsh-Hadamard matrix (each symbol goes to each subcarrier),
all $\tilde{G}_N^{(i)}$ are identical to the G_N up to the row permutations.



Relation to the bent functions

- A Boolean function of n variables is called ***bent***, if its Walsh transform coefficients are all of equal magnitude $(2^{\frac{n}{2}})$
- The corresponding truth vector is a ***bent sequence***.
- If $N = 4^k$ (n is even), then D_N as well as $\tilde{D}_N^{(i)}$, $i = 1, 2, \dots, n$ have bent sequences on the main diagonal.
- As a matter of fact, for any bent sequence \vec{f} , $H_N \cdot \text{diag}\{\vec{f}\}$ will give an orthogonal spreading matrix, providing lower PAPR, than H_N for the resulting signal.

Experimental results



If H_N is the Walsh matrix, G_N just slightly outperforms $H_N \cdot \text{diag}\{\vec{f}\}$

If H_N is other HWP matrix, G_N performs significantly better



Advantages of our approach

- We provide a constructive way of spreading matrix generation, which will necessarily provide low PAPR
- It is valid for both odd and even n , while bent functions are defined for even n only
- It gives good results for any HWP matrix H_N
- Our approach is naturally generalized to complex-valued spreading, and multiple-valued logic.



Complex-valued generalizations(1)

- Complex-valued counterpart of the Walsh transform – **Chrestenson transform**. The entries of the transform matrix are complex numbers, taking p - th roots of unity

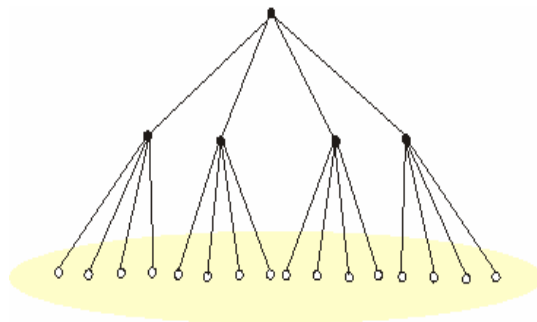
$$H_{N,p}(j+1, k+1) = e^{\frac{2\pi i}{p} C(j,k)}, \quad C(j,k) = \sum_{\eta=0}^{n-1} j_{\eta} k_{nn-1-\eta}$$

where $N = p^n$; $j, k \in \{0, 1, \dots, p^n - 1\}$; $n \geq 1$;

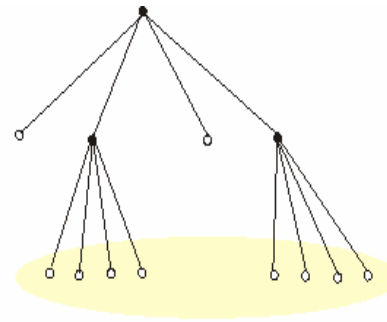
(j_1, j_2, \dots, j_n) and (k_1, k_2, \dots, k_n) are p -ary expansions of j and k respectively

Complex-valued generalizations(2)

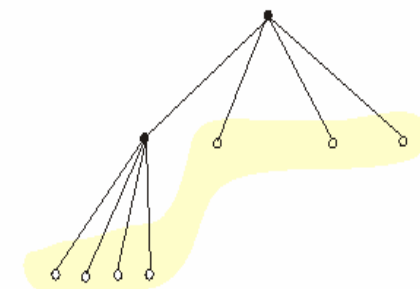
- The decomposition tree is p – ary in this case
- Wavelet packet approach allows to take any of its pruned subtrees to form the orthogonal basis



(a)



(b)



(c)

- (a) - complete decomposition
(b) - some other decomposition
(c) - complex-valued Haar (generalized Haar) decomposition

Complex-valued spreading and multiple-valued logic(1)

- Complex-valued spreading matrix:

$$G_{N,p} = H_{N,p} \cdot D_{N,p}$$

$H_{N,p}$ - matrix, resulting from any wavelet-packet type decomposition between Chrestenson and complex-valued Haar

$$D_{N,p} = \begin{bmatrix} D_{\frac{N}{p},p} \cdot \Lambda^0 & \circ & \dots & \circ \\ \circ & D_{\frac{N}{p},p} \cdot \Lambda^1 & \dots & \circ \\ \vdots & \vdots & \ddots & \vdots \\ \circ & \circ & \dots & D_{\frac{N}{p},p} \cdot \Lambda^{p-1} \end{bmatrix} \quad \Lambda = \begin{bmatrix} I_{\frac{N}{p^2}} & \circ & \dots & \circ \\ \circ & \gamma_p \cdot I_{\frac{N}{p^2}} & \dots & \circ \\ \vdots & \vdots & \ddots & \vdots \\ \circ & \circ & \dots & \gamma_p^{p-1} \cdot I_{\frac{N}{p^2}} \end{bmatrix}$$

$D_{p,p}$ - ($p \times p$) identity matrix

γ_p - p -th root of unity, $\gamma_p = e^{\frac{2\pi i k}{p}}$, $k \in \{1, 2, \dots, p-1\}$ k, p are relatively prime

Complex-valued spreading and multiple-valued logic(2)

- By taking the rows \vec{f}_i of Chrestenson matrix, corresponding to the multiple-valued (p -valued) logic functions of the form

$$f_i(x) = cx_k, \quad c \in \{1, 2, \dots, p-1\}, \quad i \in \{1, 2, \dots, (p-1)n\},$$
$$k \in \{1, 2, \dots, n\}, \quad n = \log_p(N),$$

we define new diagonal matrices

$$\tilde{D}_{N,p}^{(i)}(k, k) = D_{N,p}(k, k) \cdot \vec{f}_i(k), \quad i \in \{1, 2, \dots, (p-1)n\}, \quad k \in \{1, 2, \dots, N\}$$

The new set of spreading matrices will be:

$$\tilde{G}_{N,p}^{(i)} = H_{N,p} \cdot \tilde{D}_{N,p}^{(i)}, \quad i \in \{1, 2, \dots, (p-1)n\}$$

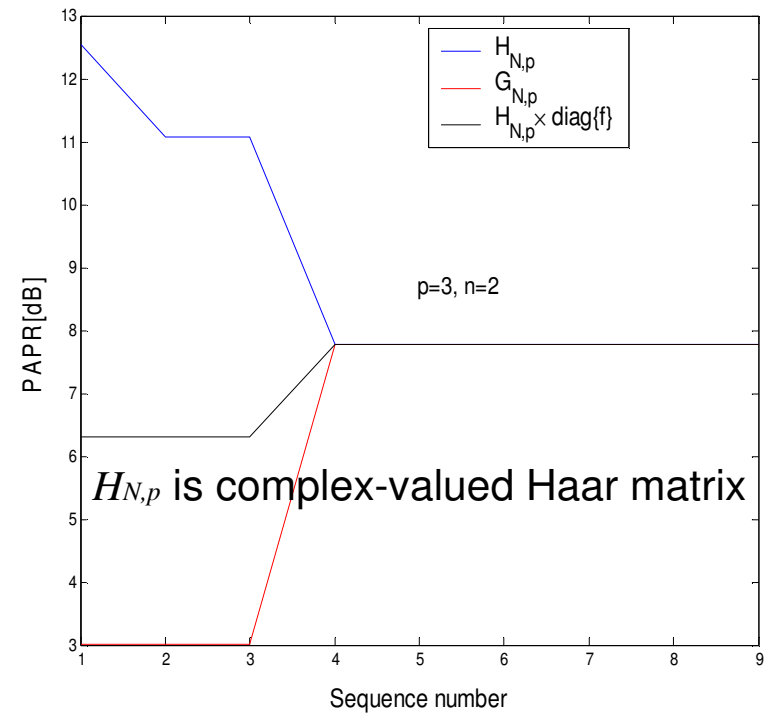
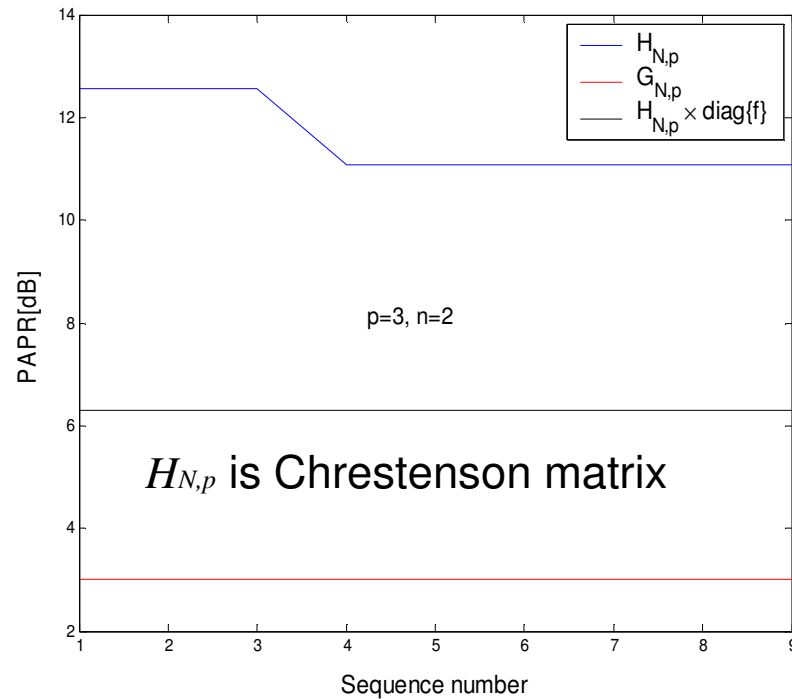
All described matrices $\tilde{G}_{N,p}^{(i)}$ as well as $G_{N,p}$, constructed earlier, will result in a low PAPR of the resulting transmitted signal

Relation to the complex-valued bent functions

By complex-valued bent function we call the function, whose Chrestenson transform coefficients are all of equal magnitude

- Only a sequence of length $N = p^{2k}$ can be complex-valued bent
- If $N = p^{2k}$, then the diagonals of $D_{N,p}$ and $\tilde{D}_{N,p}^{(i)}$ are complex-valued bent sequences.
- As in the binary case, for any complex-valued bent sequence \vec{f} , spreading the matrix $H_{N,p} \cdot \text{diag}\{\vec{f}\}$ will result in much lower PAPR of the transmitted signal, than spreading with $H_{N,p}$
- The PAPR reduction is not always that good as in the case of spreading with $G_{N,p}$ or $\tilde{G}_{N,p}^{(i)}$

Experimental results



If $H_{N,p}$ is the Chrestenson matrix, $G_{N,p}$ just slightly outperforms $H_{N,p} \cdot \text{diag}\{\vec{f}\}$

If $H_{N,p}$ is complex-valued Haar matrix, $G_{N,p}$ performs significantly better



Conclusion

- The problem of spreading code design for MC-CDMA transmission has been explored from the perspective of Boolean and multiple-valued logic
- As a result, an interesting relationship between well known orthogonal transforms, Rademacher matrices, bent functions has been established
- A family of previously unknown orthogonal transforms has been derived, which can be attractive for MC-CDMA system