An Approach to Synthesis of Multiple-Valued Reversible Logic Circuits

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Overview

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Motivation

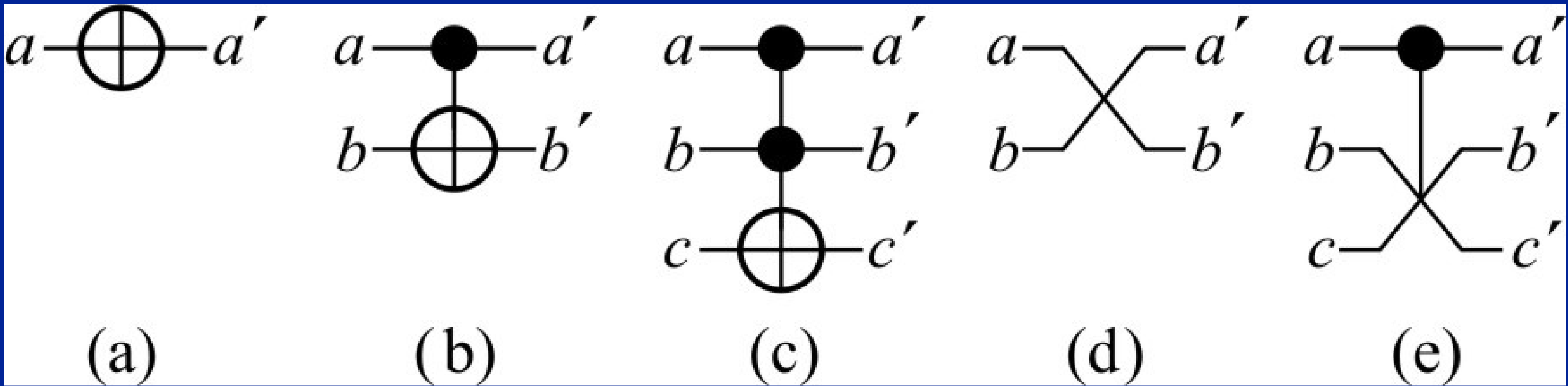
- Reversible circuits enable to reduce energy dissipation (R. Landauer 1961, C. Bennett 1973)
- Quantum processes are inherently reversible (R. Feynman 1985)
  - Logic synthesis for classical reversible circuits is a first step toward synthesis of quantum circuits
- Some important processing tasks are reversible (V.V. Shende, A.K. Prasad, I.L. Markov, J.P. Hayes, TCAD 2003)
  - Digital signal processing
  - Cryptography
  - Communication
  - Computer graphics

Reversible circuits

- A circuit (a gate) is reversible iff it realizes a bijective mapping of inputs vectors into output vectors of a truth table of the circuit (gate)
  - # inputs = # outputs
  - reversible circuit consists only of reversible gates
  - fan-out of each output = 1
  - reversible circuits are cascade circuits
- Realization of arbitrary circuits (including irreversible) sometimes requires
  - creation of additional output wires („garbage”)
  - application of constant signals to some inputs
  - application of temporary storage (i.e. wires that can be changed during computation, but must be restored by end of the computation)

Basic reversible gates and libraries

- (a) NOT (N) — $a' = 1 \oplus a$
- (b) CNOT (C) — $a' = a$, $b' = a \oplus b$
- (c) Toffoli (T) — $a' = a$, $b' = b$, $c' = c \oplus ab$
- (d) SWAP (S) — $a' = b$, $b' = a$
- (e) Fredkin (F) — $a' = a$, if $a = 0$ then $b' = b$, $c' = c$, if $a = 1$ then $b' = c$, $c' = b$



Basic gate libraries: NCT, NCTS, NCTSF

Previous work

- K. Iwama, Y. Kambayashi, S. Yamashita DAC'02
  - equivalent transformations of reversible circuits
  - canonical form
  - no synthesis algorithm
- V.V. Shende, A.K. Prasad, I.L. Markov, J.P. Hayes ICCAD'02
  - synthesis of optimal 3-wire reversible circuits
  - can be used to generate libraries of small optimal ckts.
  - not implemented for larger functions
- D.M. Miller, D. Maslov, G. Dueck DAC'03, ISMVL'04
  - 2-stage transformation-based algorithm implemented (called here the DMM algorithm)
- A. Agrawal, N.K. Jha DATE'04
  - a synthesis algorithm based on PPRM expressions

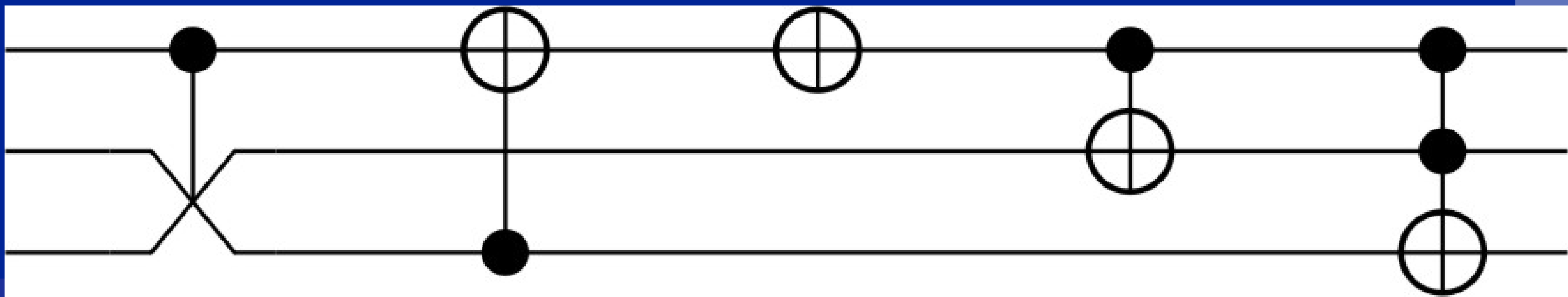
Assumptions

- Reversible specifications that can be realized without additional wires (as in the previous algorithms)
- Libraries: NCT, NCTS, NCTSF
- Cost of a circuit is its gate count
- Incremental approach to improve results of the first stage of the DMM algorithm
- At each step of the new algorithm the selection of which gate to add next is guided by minimizing the complexity of the remainder function
- New complexity measure based on using decision diagrams to represent a reversible function (instead of using truth tables or PPRM expressions, as in the previous algorithms)

New complexity measure (1)

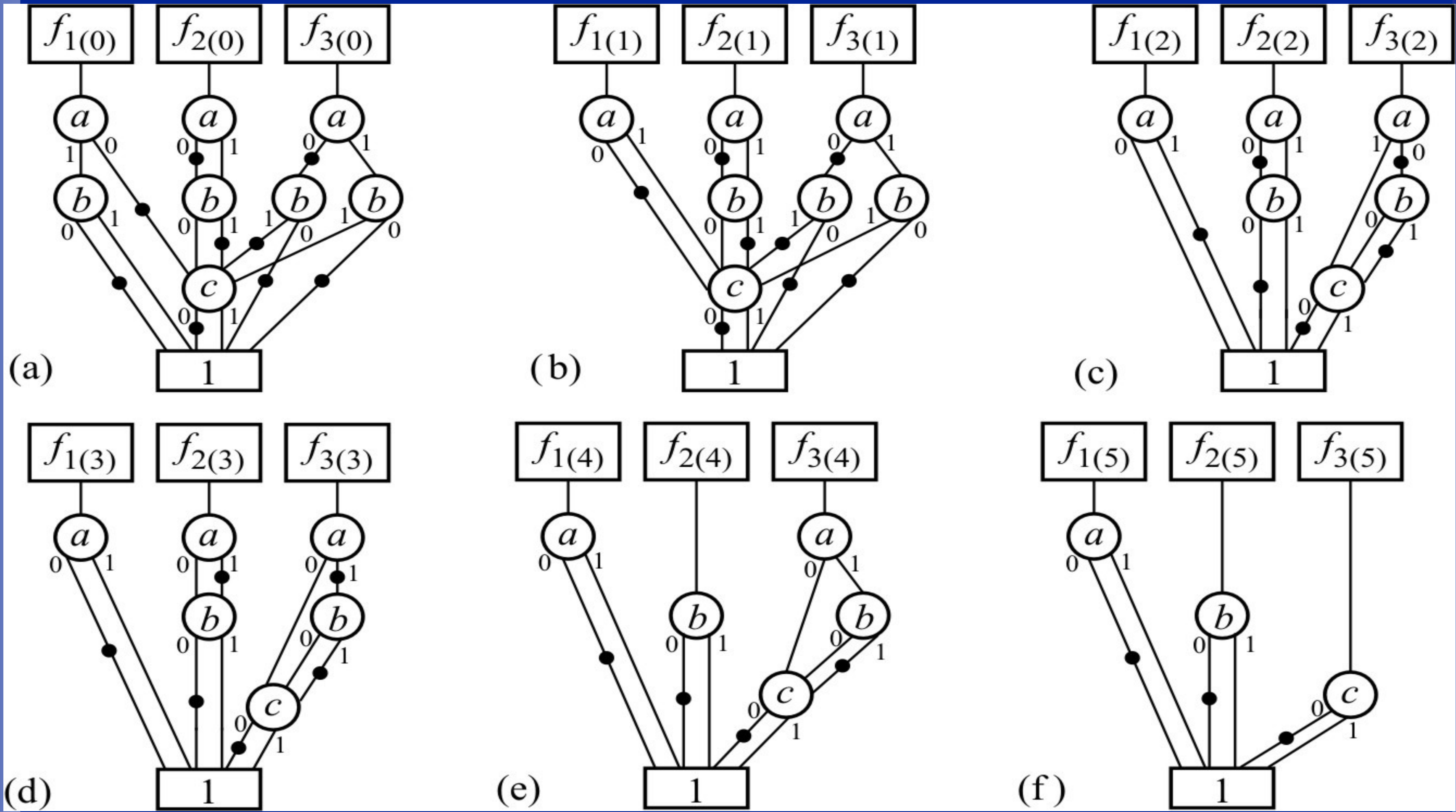
- **Definition** The complexity measure of a completely specified n-input n-output reversible Boolean function f is equal to $D(f) = s(f) - n$, where $s(f)$ denotes the number of non-terminal nodes in the reduced ordered shared binary decision diagram (SBDD) of f with complemented edges.
- **Remarks**: 1) D(identity function) = 0

  2) SBDDs with natural order of inputs
- **Example** of an optimal circuit for NCTSF library:



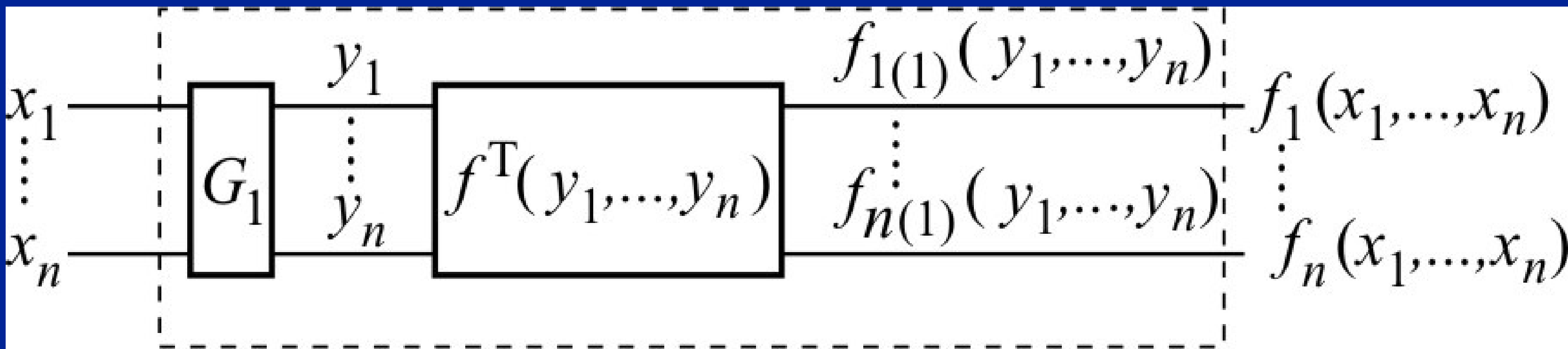
New complexity measure (2)

- SBDDs of remainder functions for the example circuit:

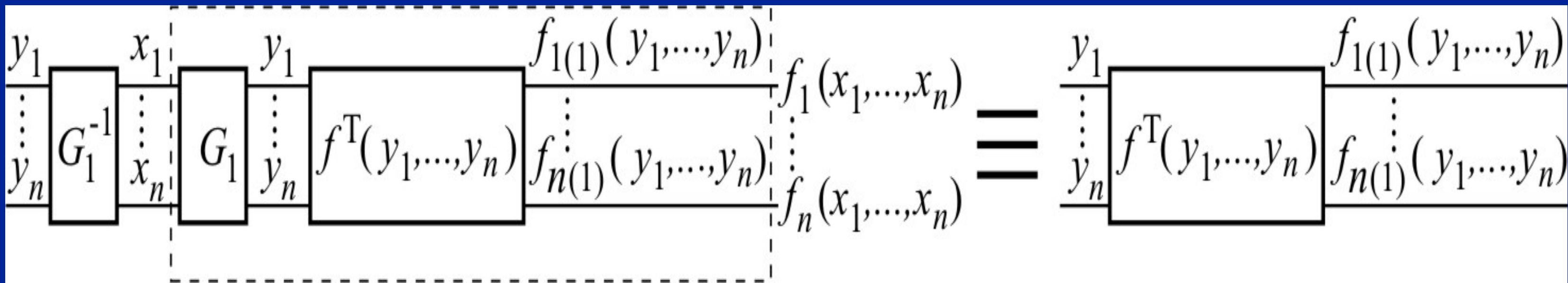


New heuristic algorithm

- Sketch of one step of our algorithm
  - for each gate construct SBDD of the remainder function
  - select gates for which the complexity measure of the remainder function is minimal (if there is more than one such gate, proceed with all of them)
- Speeding-up improvements in our algorithm
  - functions with small $D(f)$ can be implemented very fast
  - moving SWAP gates decreases # of gates considered



A way to determine a reminder function

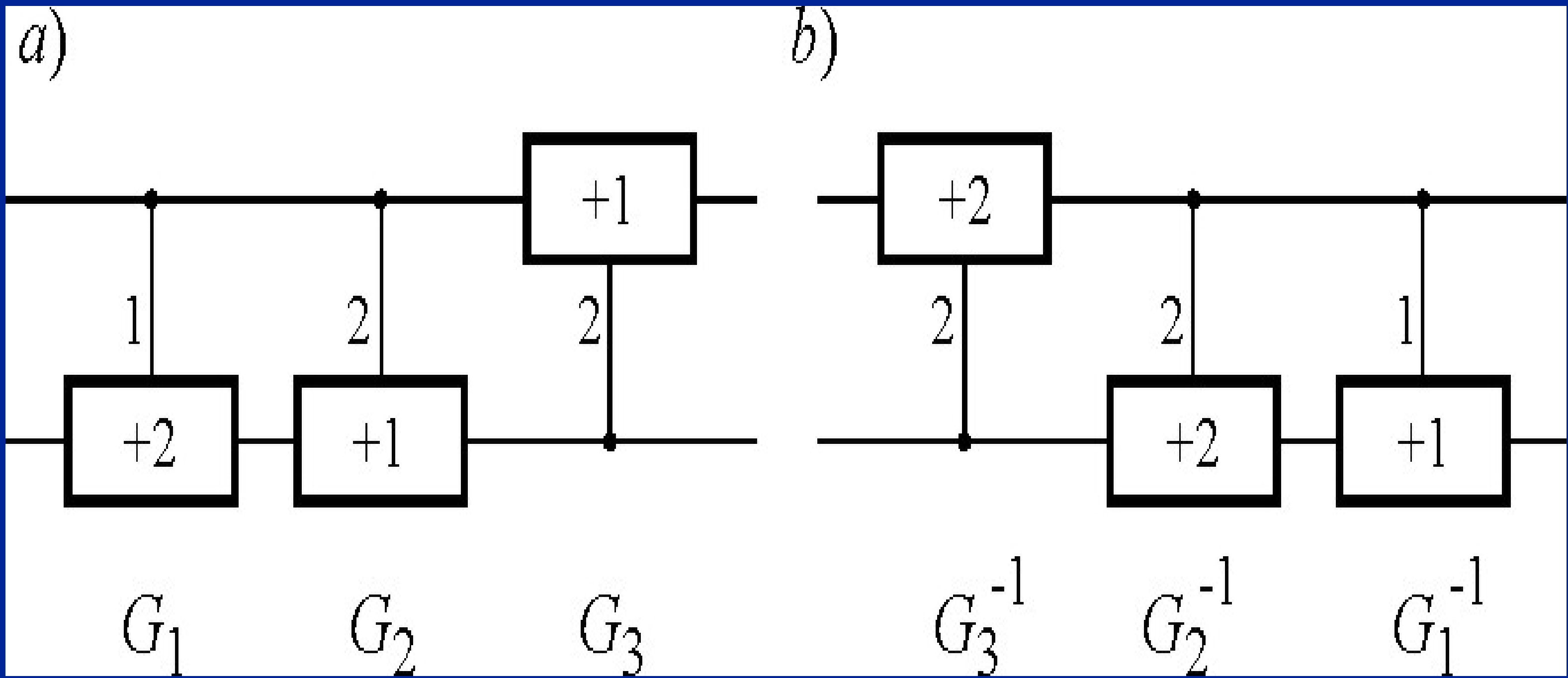


Experimental results

- Promising results in comparison with the results obtained for the DMM algorithm in the following cases:
  - synthesis of 3-wire functions
  - two libraries: NCTS and NCTSF
  - one-directional and bi-directional versions of the algorithms
  - best results of the two one-directional algorithms

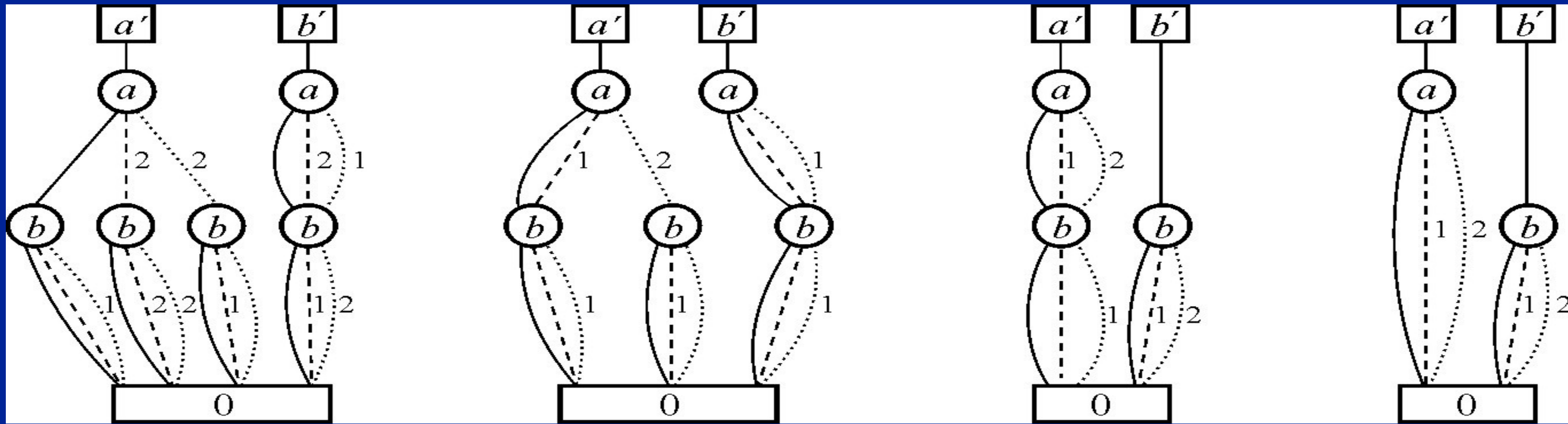
Example MVL circuits

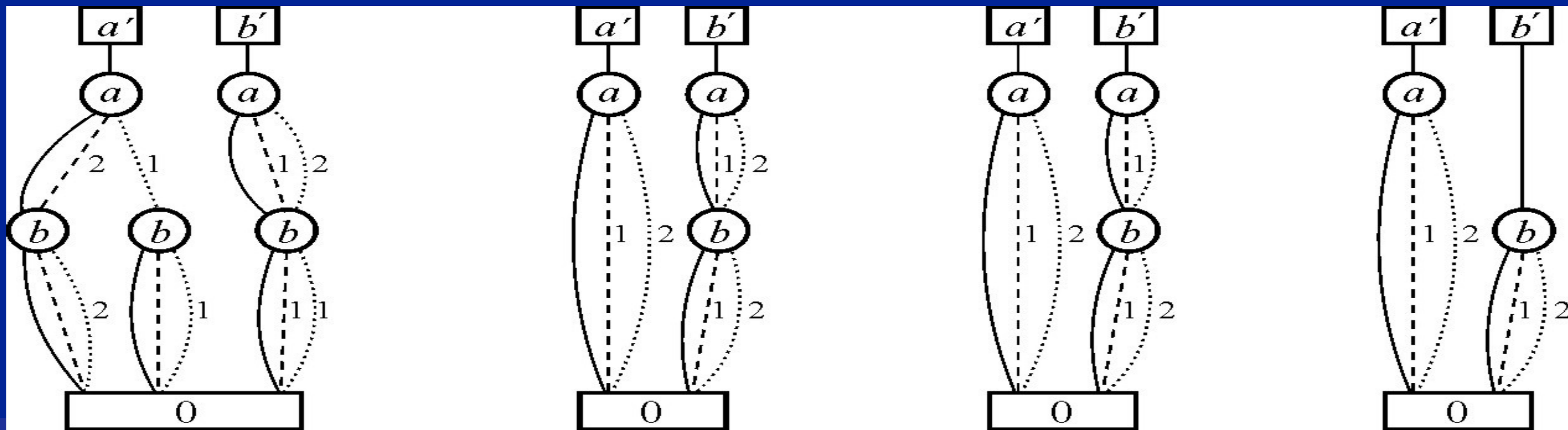
- Two ternary reversible circuits (the circuit shown in Figure *a* is the inverse of the circuit shown in Figure *b*).
- All the gates in these circuits are controlled gates corresponding to quantum realizable gates.



CSMDDs of reminder functions

- *Cyclic negations:* $C_k(x) = (x+k) \bmod p,\ 0 < k < p$





Conclusions and future work

- Our algorithm has better potential than previously reported algorithms due to using decision diagrams as a representation of reversible functions (instead of truth tables or PPRM expressions)
- Our algorithm works for:
  - arbitrary libraries
  - arbitrary cost functions
- Future work
  - modifications of our complexity measure to incorporate more data for obtaining better efficiency of the synthesis algorithm