Encoding of network of FSMs guided by Measure of Information Relationship

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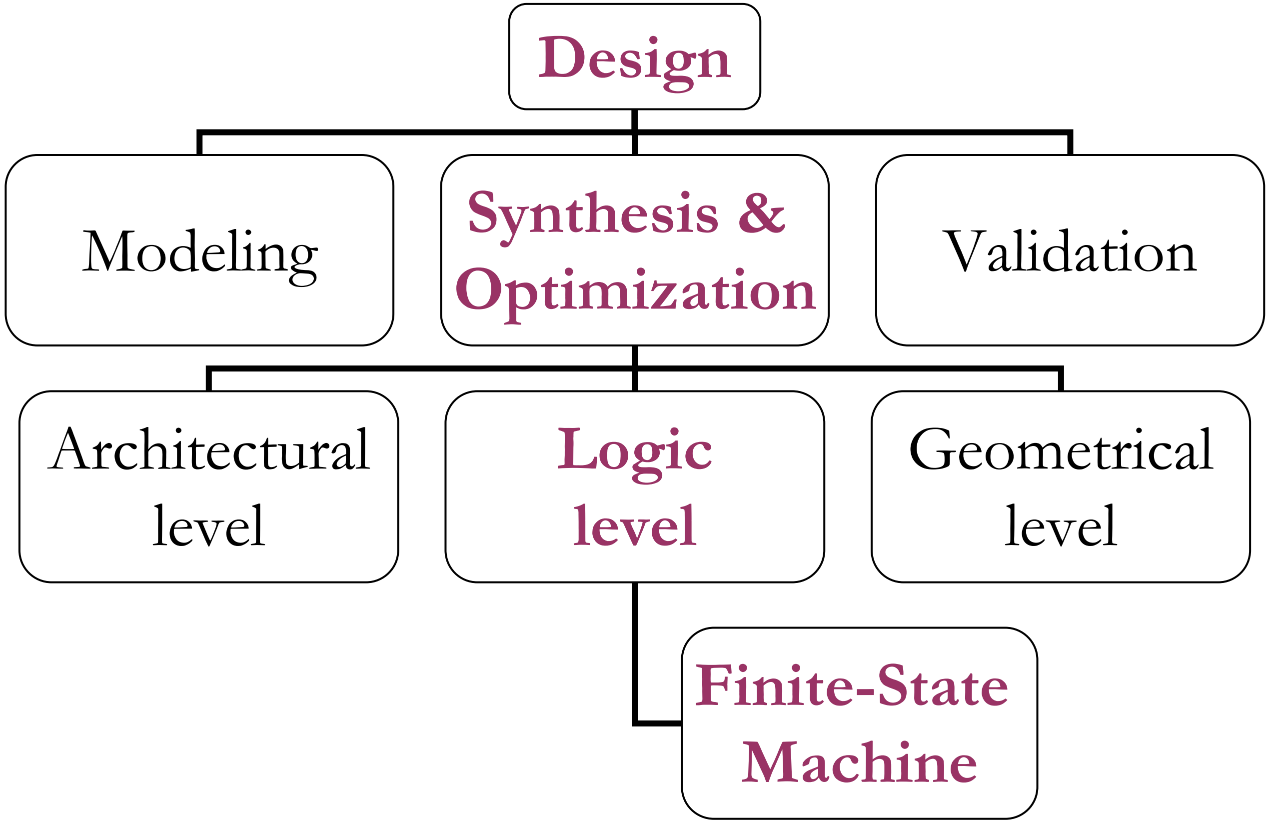
Computer-Aided Design



FSM Synthesis & Optimization

State minimization	State encoding
Relevant optimization technique – FSM decomposition	Binary representation of the states of an FSM

System of Partitions

$\boldsymbol{S}$ – a set of the states of a decomposable machine

$\{\pi(\boldsymbol{S})\}$ – the system of partitions on the set $\boldsymbol{S}$,

where $\pi(\boldsymbol{S})=\{\boldsymbol{B}_a(\boldsymbol{S})\}$ such that

$\boldsymbol{B}_a \cap \boldsymbol{B}_b = \varnothing$ for $\boldsymbol{a} \neq \boldsymbol{b}$ and $\cup\{\boldsymbol{B}_a\} = \boldsymbol{S}$

Complete System of Partitions

A system of partitions $\{\pi_i(S)\}$ is a **complete system of partitions** if and only if

$$\prod_{i=1}^{n} \pi_i(S) = \pi_0$$

$$S = \{s_1, s_2, \ldots, s_m\}, \;\; \pi_0 = \left\{\overline{s_1}, \overline{s_2}, \ldots, \overline{s_m}\right\}$$

Example

Complete System of Partitions

$$
\left\{
\begin{array}{l}
\pi_1 = \left\{\overline{1,2};\overline{3,4,5};\overline{6,7};\overline{8}\right\} \\
\pi_2 = \left\{\overline{1};\overline{2,3,4,6,8};\overline{5,7}\right\} \\
\pi_3 = \left\{\overline{1,3,7};\overline{2,4,8};\overline{5,6}\right\}
\end{array}
\right\}
$$

$$
\pi_0 = \left\{\overline{1},\overline{2},\overline{3},\overline{4},\overline{5},\overline{6},\overline{7},\overline{8}\right\}
$$

State Encoding

Given a set of symbols $S=\{s_1, s_2, \ldots, s_m\}$

and an integer k,

a **binary encoding** of S

is one-to-one mappings

$e: S \rightarrow \{0,1\}^k$

Face Constraint

Given a set of symbols *S*, a **face constraint** C_f is a subset $S' \subseteq S$ specifying that the symbols in ***S'*** are to be assigned to one face (or sub-cube) of a binary ***k***-dimensional cube, without any other symbol sharing the same face

Face Hypercube Embedding

To find the minimum k – **code length**

and related e – **encoding**:

$$S \longrightarrow \{0,1\}^k$$

such that C – **set of constraints**
is satisfied

Decomposition Constraints

S – the set of the states of a decomposable machine

$\{\pi\}$ – a complete system of partitions

C – a set of decomposition constraints:

$$C = \left\{ B_{\pi_i}^{j} \mid i \in \{1, \ldots, n\},\ j \in \{1, \ldots, |\pi_i|\} \right\}$$

$B_{\pi_i}^{j}$ – the blocks of the partitions from $\{\pi\}$

Encoding Partition

An **encoding partition** is a two-block partition of a subset of the symbols to be encoded:

$$g_{\pi_i} = \left\{ B_{\pi_i}^{(0)}, B_{\pi_i}^{(1)} \mid B_{\pi_i}^{(0)} = \bigcup_{0} B_{\pi_i}^{j}; B_{\pi_i}^{(1)} = \bigcup_{1} B_{\pi_i}^{j} \right\}$$

We form the two-block partitions for each partition from the complete system of partitions separately. It means that we have a limited choice in generating a set of encoding partitions

Example

Set of Encoding Partitions

$$g_{\pi_1}^{1} = \{\overline{1,2};\overline{3,4,5,6,7,8}\}$$

$$g_{\pi_1}^{2} = \{\overline{1,2,3,4,5};\overline{6,7,8}\}$$

$$g_{\pi_1}^{3} = \{\overline{1,2,3,4,5,6,7};\overline{8}\}$$

$$g_{\pi_1}^{4} = \{\overline{1,2,6,7};\overline{3,4,5,8}\}$$

$$g_{\pi_1}^{5} = \{\overline{1,2,8};\overline{3,4,5,6,7}\}$$

$$g_{\pi_1}^{6} = \{\overline{1,2,6,7,8};\overline{3,4,5}\}$$

$$g_{\pi_1}^{7} = \{\overline{1,2,3,4,5,8};\overline{6,7}\}$$

$$g_{\pi_2}^{1} = \{\overline{1,2,3,4,6,8};\overline{5,7}\}$$

$$g_{\pi_2}^{2} = \{\overline{1};\overline{2,3,4,5,6,7,8}\}$$

$$g_{\pi_2}^{3} = \{\overline{1,5,7};\overline{2,3,4,6,8}\}$$

$$g_{\pi_3}^{1} = \{\overline{1,2,3,4,7,8};\overline{5,6}\}$$

$$g_{\pi_3}^{2} = \{\overline{1,3,5,6,7};\overline{2,4,8}\}$$

$$g_{\pi_3}^{3} = \{\overline{1,3,7};\overline{2,4,5,6,8}\}$$

Partition & Information

A partition π on a set S is an equivalent of the information about this set to accuracy of a block of the partition π

$$\pi(S) = \left\{ \overline{B_1(S)}, \overline{B_2(S)}, \ldots, \overline{B_j(S)}, \ldots \right\}$$

Set System & Information

A set system ***SS*** on a set ***S*** is a collection of nonempty subsets of ***S*** such that:

$$SS(S)=\{B(S)\} \quad \bigcup\{B(S)\}=S$$

A certain set system gives information about the elements of ***S*** with limited precision to the compatibility (equivalence) class

System of Partitions → Set System

All blocks of a complete system of partitions $\{\pi\}$

on the set ***S*** form a set system ***SS*** on the set ***S***:

$$SS(S) = \left\{ \begin{array}{l} \overline{B_1^1(S)}, \overline{B_2^1(S)}, \ldots, \overline{B_j^1(S)}, \ldots, \\ \overline{B_1^2(S)}, \overline{B_2^2(S)}, \ldots, \overline{B_j^2(S)}, \ldots, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \overline{B_1^n(S)}, \overline{B_2^n(S)}, \ldots, \overline{B_j^n(S)}, \ldots \end{array} \right\}$$

Information Relationship

$$SS(s) \cdot \pi(s) = \left\{ \begin{array}{l} B_{SS}(s) \cap B_{\pi}(s) \mid \\ B_{SS}(s) \in SS(s) \wedge B_{\pi}(s) \in \pi(s) \end{array} \right\}$$

The product of a set system and a partition represents joint information about the elements of ***S*** that is provided by the relations together

Example

Information Relationship

SS(***S***) – the set system on the set ***S***:

$$SS(S)=\{\overline{1,2};\overline{1,3,7};\overline{2,4,8};\overline{3,4,5};\overline{5,6};\overline{6,7}\}$$

π(***S***) – the partition on the set ***S***:

$$\pi(S)=\{\overline{1,3,4,5};\overline{2};\overline{6,7,8}\}$$

Information relationship between ***SS***(***S***) and ***π***(***S***):

$$SS(S)\cdot\pi(S)=\{\overline{1};\overline{2};\overline{1,3};\overline{4};\overline{3,4,5};\overline{5};\overline{6};\overline{7};\overline{8}\}$$

$$SS(S) < SS(S)\cdot\pi(S)$$

Information Relationship

The partial ordering relation denotes the fact that if

$$SS_1(S) \leq SS_2(S)$$

then SS_1 provides the same or more information about elements of S than SS_2

An elementary information describes the ability to distinguish a certain single symbol s_i from another symbol s_j, where s_i, $s_j \in S$ and $s_i \neq s_j$

Heuristic Approach

The base of the heuristic approach is selecting such

encoding partition $g_{\pi_i}(s)$ that

$$\max \left| ss(s) \cdot g_{\pi_i}(s) \right|$$

The aims of the heuristic approach are to satisfies

the decomposition constraints and to give the

minimum code length

Example

Information Relationship Measure

$$SS(S)=\{\overline{1,2};\overline{3,4,5};\overline{6,7};\overline{8};\overline{1};\overline{2,3,4,6,8};\overline{5,7};\overline{1,3,7};\overline{2,4,8};\overline{5,6}\}$$

$$g^{1}_{\pi_1}=\{\overline{1,2};\overline{3,4,5,6,7,8}\}$$

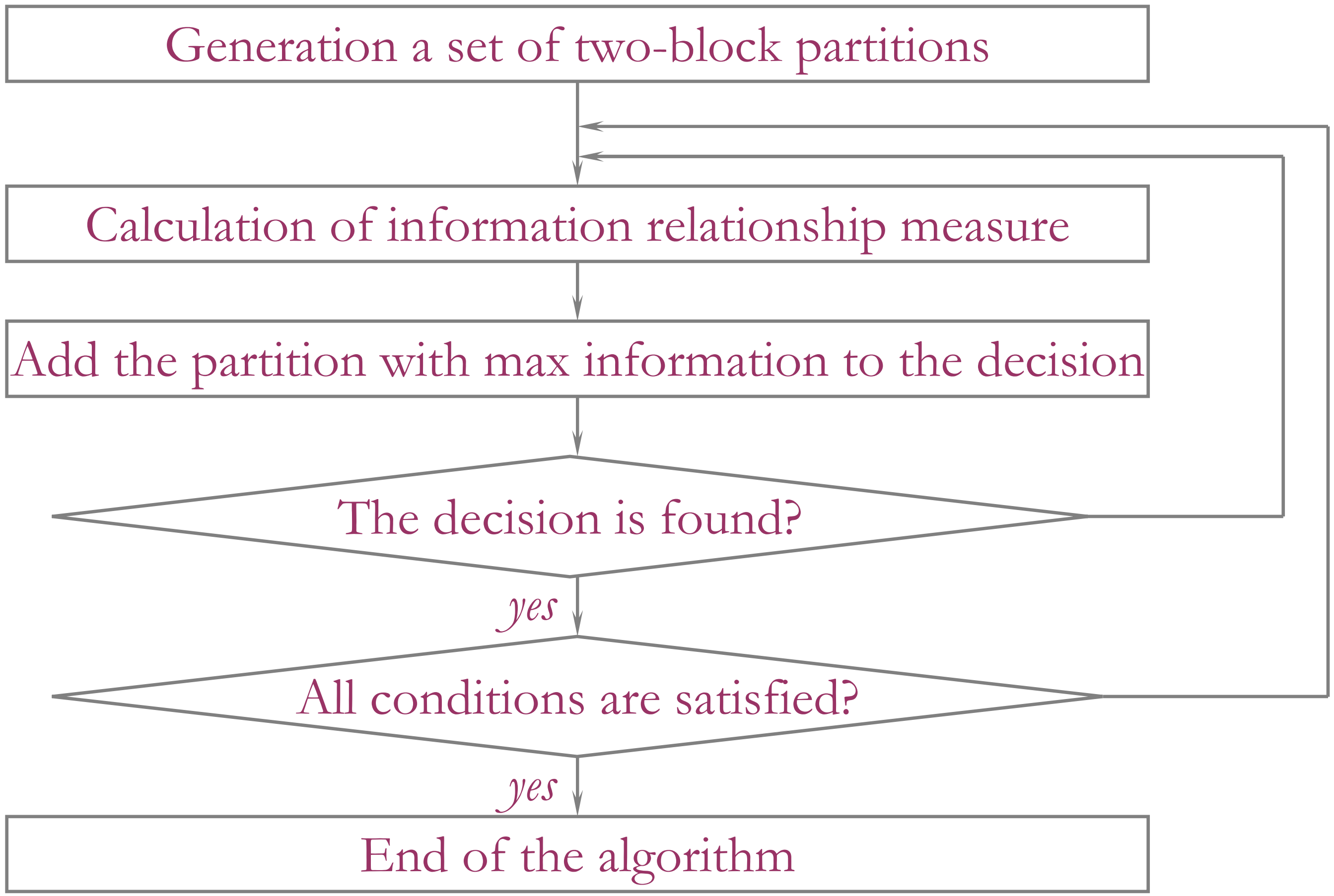
$$g^{2}_{\pi_1}=\{\overline{1,2,3,4,5};\overline{6,7,8}\}$$

$$\left|SS(s)\cdot g^{1}_{\pi_1}\right|=13$$

$$\left|SS(s)\cdot g^{2}_{\pi_1}\right|=15$$

$$SS'(S)=\{\overline{1};\overline{1,2};\overline{2};\overline{1,3};\overline{2,4};\overline{3,4};\overline{3,4,5};\overline{5,6};\overline{6,7};\overline{7};\overline{6,8};\overline{8}\}$$

Encoding Algorithm



Example

Encoding and Solution Matrixes

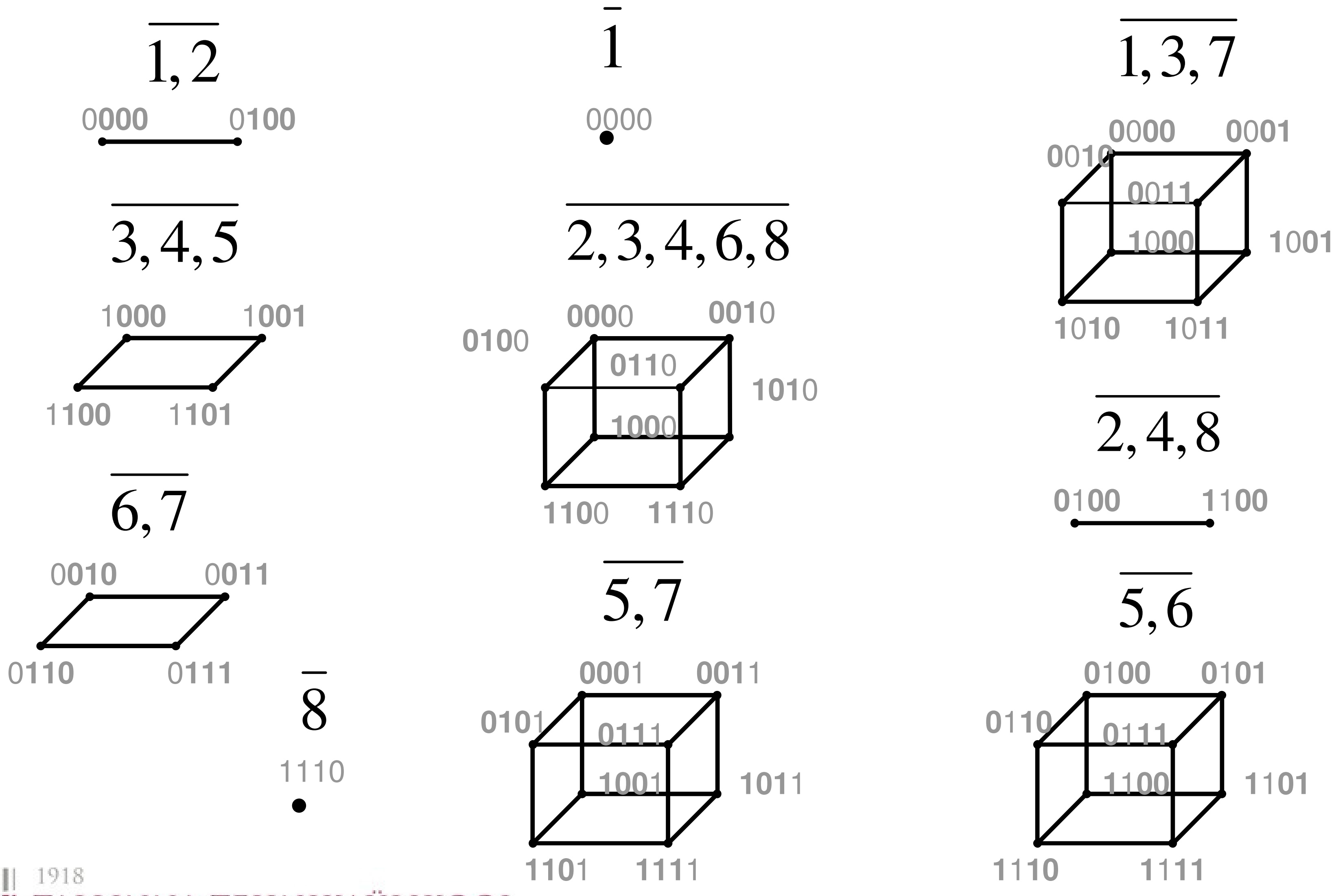
States of the decomposable machine

$$E^{new} = \begin{bmatrix} 0\,0\,0\,0 \\ 0\,1\,0\,0 \\ 1\,0\,0\,0 \\ 1\,1\,0\,0 \\ 1\,1\,0\,1 \\ 0\,1\,1\,0 \\ 0\,0\,1\,1 \\ 1\,1\,1\,0 \end{bmatrix}$$

$$s_N = \begin{bmatrix} 0\,-\,0\,0 \\ 1\,-\,0\,- \\ 0\,-\,1\,- \\ 1\,1\,1\,0 \\ 0\,0\,0\,0 \\ -\,-\,-\,0 \\ -\,-\,-\,1 \\ -\,0\,-\,- \\ -\,1\,0\,0 \\ -\,1\,-\,- \end{bmatrix}$$

States of the network of machines

Boolean Cubes



4-D Boolean Cube

{1,2}

{3,4,5}

{6,7}

{8}

{1}

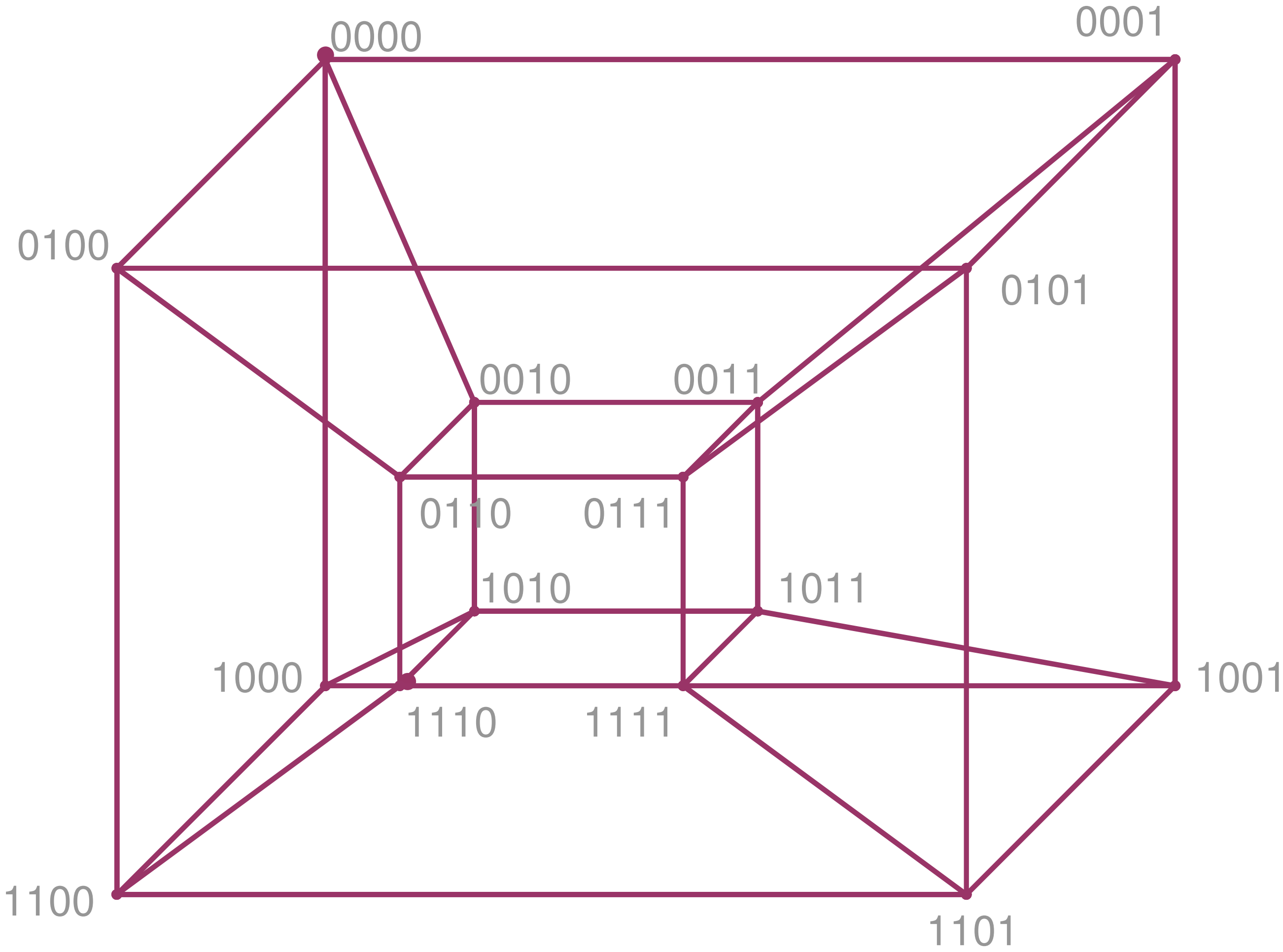
{2,3,4,6,8}

{5,6}

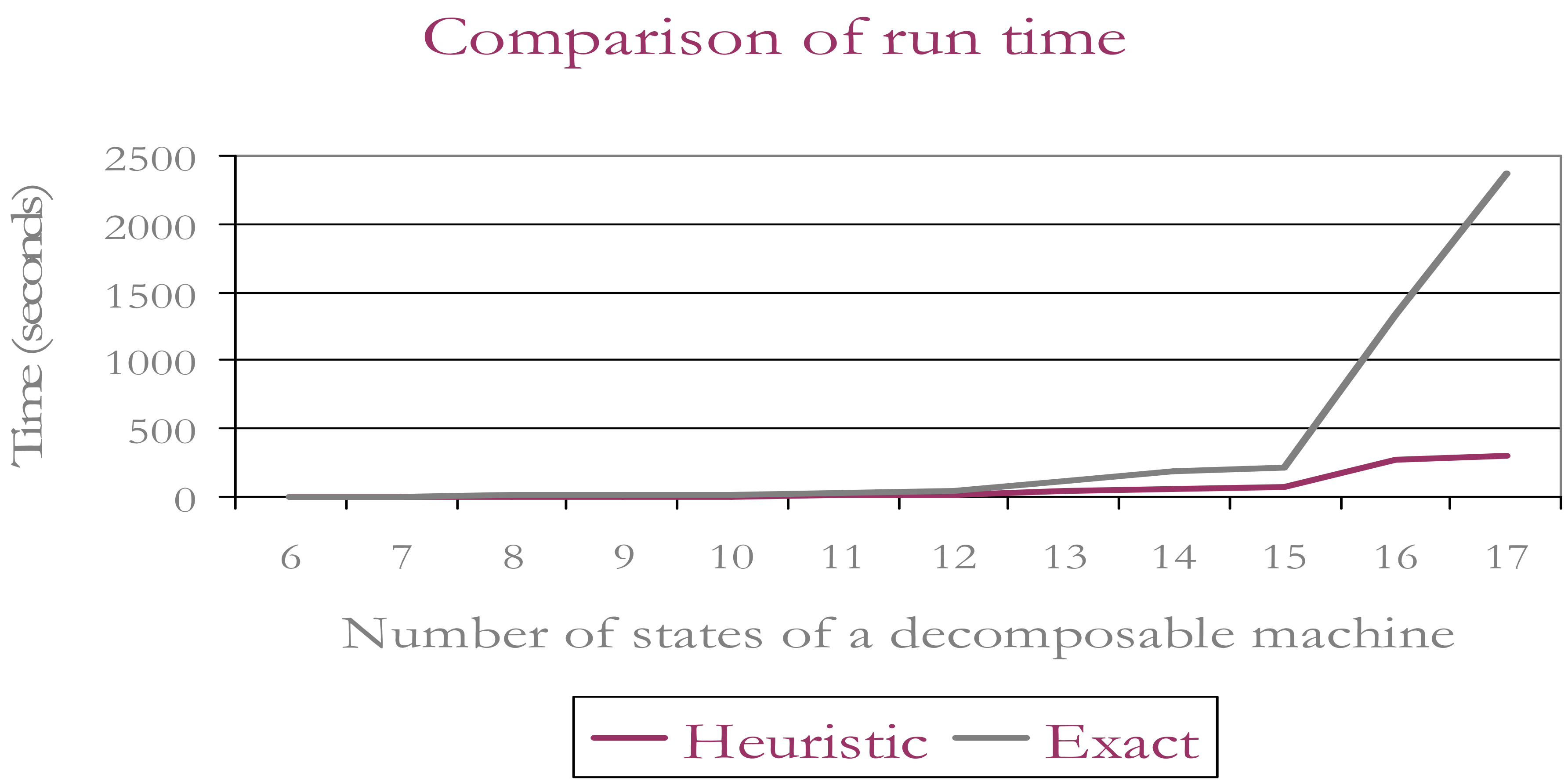
{1,3,7}

{2,4,8}

{5,7}



Algorithm Performance



Practical Application

Project
DILDIS

Tallinn University of Technology &
Ilmenau Technical university

http://www.pld.ttu.ee/dildis

Encoding of network of FSMs

http://www.pld.ttu.ee/dildis/automata/applets/dsa